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CRREL REPORT 88-6

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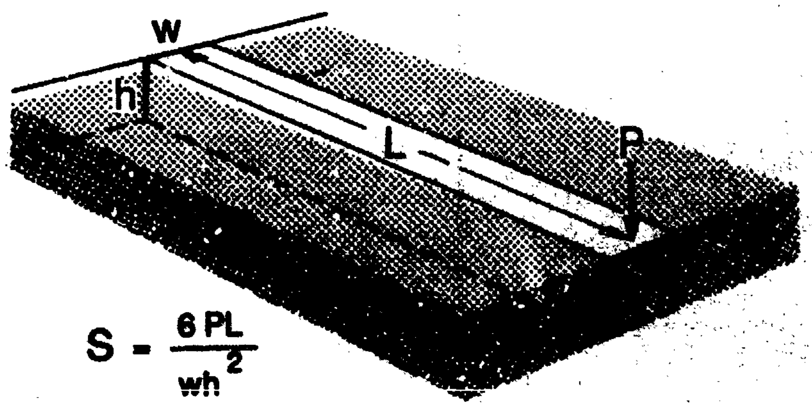
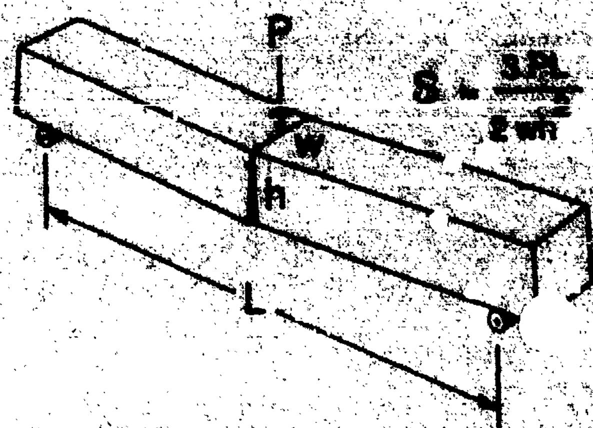
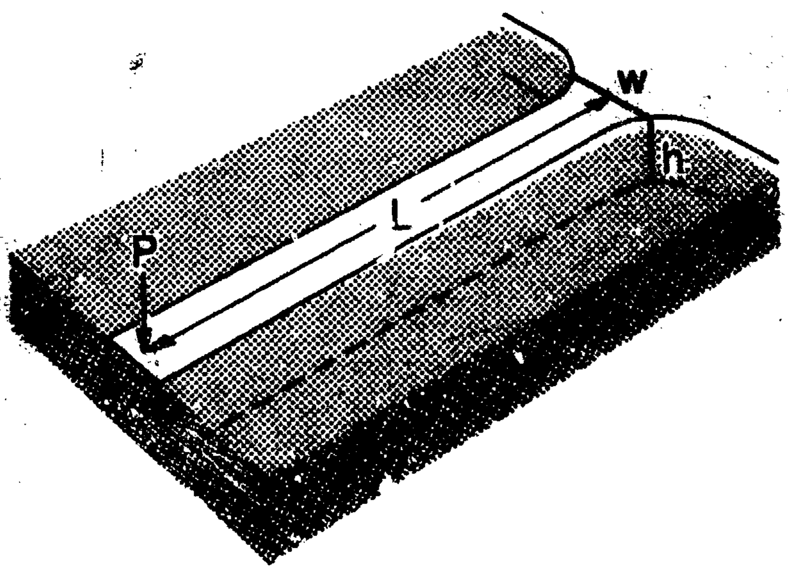
Temperature and structure dependence of the flexural strength and modulus of freshwater model ice







Grain size
c-axis orientation
Temperature
Temperature gradient
Stress concentration

CRREL Report 88-3

June 1988

Temperature and structure dependence of the flexural strength and modulus of freshwater model ice

Anthony J. Gow, Herbert T. Ueda, John W. Govoni and John Kalafut

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This report presents results of small beam testing conducted in a test tank on ice corresponding in structure to the two major ice types, S1 and S2, encountered in lake ice sheets. Tests of 730 beams in the temperature range −1 to −19°C showed that macrocrystalline (S1) and columnar (S2) ice differ appreciably in their flexural characteristics, and that these differences are attributable to variations in the size and orientation of the crystals in the ice and the thermal condition of the beams. Parallel testing of cantilever and simply-supported beams indicated a virtual non-dependence of flexural strength on the temperature of the fiber in tension. It was also determined that the sharply terminated corners of conventional cantilever beams are a source of appreciable stress concentration that can reduce the intrinsic flexural strength by as much as one-half, but which, in most cases, can be substantially relieved by drilling holes at the beam roots. Overall, flexural strengths did not exceed 1200 kPa for cantilever beams or 1650 kPa for simply supported beams tested in parallel with cantilever beams. The highest flexural strengths were measured on isothermal simply supported beams of S2 ice tested with the top surface in tension, with average strengths for such ice increasing from 1650 kPa at −1°C to nearly 2600 kPa at −19°C. Beams

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19. Abstract (cont'd)

made to fail with bottom in tension tested about 35% weaker because of the greatly increased size of crystals in the bottom of S2 ice sheets. Beams of S1 ice yielded flexural strengths midway between those measured on S2 ice. This behavior, which occurs despite the fact that crystal size in S1 ice is always very much larger than in the coarsest-grained S2 ice, is attributed to crystal orientation effects in which the dominant vertical c-axis structure that characterizes S1 ice forces tensile failure to propagate in the hard-fail plane of the crystals. It was also determined that temperature gradients decreased flexural strengths in simply supported beams, by as much as 45–50% at the largest temperature gradients, compared to isothermal simply supported beams tested at the same ambient air temperatures. Strain modulus measurements showed little dependence on either the temperature of the beam or its flexural strength, with average values ranging from 5–7 GPa for cantilever and parallel simply supported beams, and 6–8 GPa for isothermal simply supported beams.

PREFACE

This report was prepared by Dr. Anthony J. Gow, Research Geologist, and John W. Govoni, Physical Science Technician, Snow and Ice Branch, Research Division; Herbert T. Ueda, Mechanical Engineer, and John Kalafut, Electronics Engineer, Engineering and Measurement Services Branch, Technical Services Division, U.S. Army Cold Regions Research and Engineering Laboratory. The study was funded under DA Project 4A161102 AT24, *Research on Snow, Ice and Frozen Ground*; Task SS, *Combart Service Support*; Work Unit 002, *Physical Properties of Snow and Ice*.

This report was technically reviewed by Dr. John Dempsey (Clarkson University) and Dr. Jean-Claude Tatinclaux (CRREL). The authors wish to thank Betsy Holt for tabulation, compilation and organization of the large quantity of raw data and Lynne Martel for preparing the appendix tables. The assistance of David Fisk, who helped with some of the testing, and of the word processing, photo services, drafting and editing sections is also appreciated.

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Temperature and Structure Dependence of the Flexural Strength and Modulus of Freshwater Model Ice

ANTHONY J. GOW, HERBERT T. UEDA,
JOHN W. GOVONI AND JOHN KALAFUT

INTRODUCTION

Previous investigations by Gow et al. (1978) of the flexural strength of large beams of lake ice have indicated that the strength of the ice depends appreciably on its crystalline composition and temperature. This work, carried out mainly on S1 ice sheets composed of macrocrystalline ice, overlain by fine-grained snow ice, showed two things. First, that simply supported beams yielded much higher flexural strengths than the same beams tested in the cantilever mode (this behavior was attributed to the existence of sizable stress concentrations at the sharp-cornered roots of cantilever beams; only in isothermal, structurally degraded ice did this effect disappear). Second, fine-grained ice at the top of the ice sheet reacted more strongly in tension than coarser-grained ice at the bottom. The ratio of strength for the top in tension to that for the bottom in tension occasionally exceeded 2.0, but averaged between 1.2 and 1.6, depending on the temperature of the ice sheet. This work on large ice beams has now been extended to studies of freshwater model ice under laboratory-controlled conditions using a combination of cantilever and simply supported beams to ascertain the dependence of the flexural behavior of the ice on its crystalline structure and temperature.

Several years of observations of the crystalline structure of ice sheets forming on a number of New England lakes indicate that only two major types of congelation ice are formed during quiet freezing of lake water.* These are 1) ice sheets composed of massive, irregularly shaped crystals exhibiting vertical or near-vertical c-axes, so-called S1 ice as defined by Michel and Ramseier (1971), and 2) ice sheets composed predominantly of vertically elongated crystals exhibiting mainly horizontally oriented c-axes, so-called S2 or columnar ice. This strong relationship of the size and shape of ice crystals to lattice orientation is an outstanding example of orientation texture in a natural setting and is discussed in greater detail in Gow (1986).

According to some researchers (e.g., Cherepanov and Kamyshnikova 1973), the thermal regime of the water as it is about to freeze is the critical determinant of orientation texture. According to Cherepanov (as cited in Lavrov 1971), S1 ice is formed when the temperature of the water beneath it is close to 4°C. However, if all the water is cooled to 0°C, S2 type ice is formed. Apart from Lavrov's own experiments, indicating that seeding or not seeding the water immediately prior to freezing might be just as important as the temperature regime of the water, little if any systematic attempt has been made to determine, experimentally, what the precise nature of the mechanisms are that control orientation texture in quietly frozen water.

TEST TANK SIMULATION

As a prelude to beam testing of freshwater model ice, a series of experiments was conducted in a refrigerated test tank at CRREL to evaluate both the effects of seeding of the water and its thermal condition on the orientation texture of ice sheets. The tank measured 7 by 7 m and was filled with water to a depth of 1.2 m. The water contained the same concentrations of dissolved solids (4–8 mS/m) as found in local lakes. A circulating pump installed in the bottom of the tank was used to cool the water column uniformly to any tempera-

* Lake ice sheets are composed typically of two major components, snow ice and lake ice. Snow ice forms by freezing of water-soaked snow on top of an existing ice sheet, whereas lake ice per se is formed by direct freezing of lake water to the underside of an ice sheet. The latter ice type is usually referred to as congelation ice.

ture below 4°C, the temperature of maximum density of fresh water. Water temperatures were measured to an accuracy of ±0.2°C with the aid of two thermocouple strings located near the edge and at the center of the tank respectively. As soon as the desired isothermal temperature between 4 and 0°C had been achieved, the pump was turned off and the air temperature of the tank lowered to -20°C to promote freezing. Freezing was initiated either by spray-seeding the surface of the water with frozen droplets (using a high-pressure nozzle directed at the ceiling of the tank) or by allowing surface crystallization to nucleate spontaneously. Crystalline texture and orientation were monitored at regular intervals during the growth of an ice sheet, mainly through examination of thin sections using a microtome technique similar to that used for lake ice (Gow 1986), sea ice (Gow and Weeks 1977) and urea-doped ice (Gow 1984) used in simulation studies of sea ice.

Water that was spray-seeded prior to freezing in the tank invariably produced S2 type ice, that is, columnar textured ice with substantially horizontal c-axes. Furthermore, this orientation texture always developed regardless of the thermal condition of the water column prior to seeding. We found the intensity of seeding to exercise some control on crystal size initially—the more intense the seeding the finer grained the ice at the top of the ice sheet. However, the thermal regime of the water column, whatever its temperature between 4 and 0°C, appeared to exercise little if any effect on crystal size at any stage of growth of an ice sheet. In all seeded ice sheets, the mean crystal cross-sectional diameter increased progressively with increasing thickness from about 1–2 mm, just below the seeded ice layer, to 6–7 mm at the bottom of an 11-cm-thick ice sheet.

We observed that unseeded or spontaneously nucleated ice growth, without exception, pro-

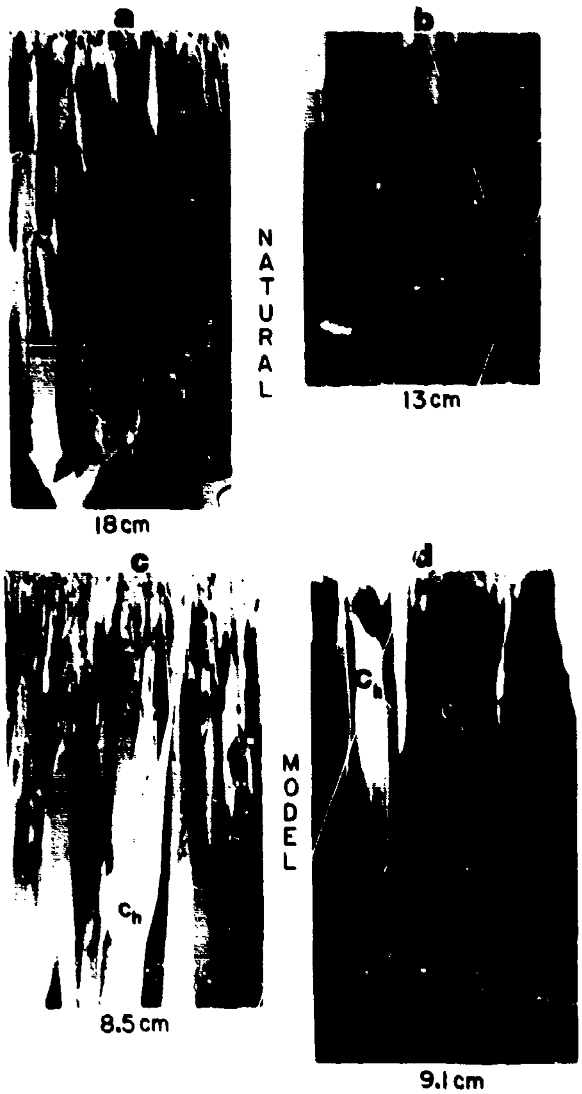


Figure 1. Vertical thin sections of crystal structure in naturally frozen lake ice (a and b) and in model freshwater ice grown in the refrigerated tank at CRREL (c and d). In the experimentally seeded ice (c), the crystals are characteristically columnar and possess substantially horizontal c-axis (C_h) orientations. In unseeded ice (d), massively sized crystals with a dominant vertical c-axis (C_v) orientation are typical; most crystals also exhibit a striated appearance. C_h type crystals may occur at the top of unseeded ice sheets but are usually eliminated rapidly by C_v type crystals. These two ice types, produced by the simple expedient of seeding or not seeding the water before freezing it, can be seen to correspond very closely with the two major ice crystal textures observed in lake ice.

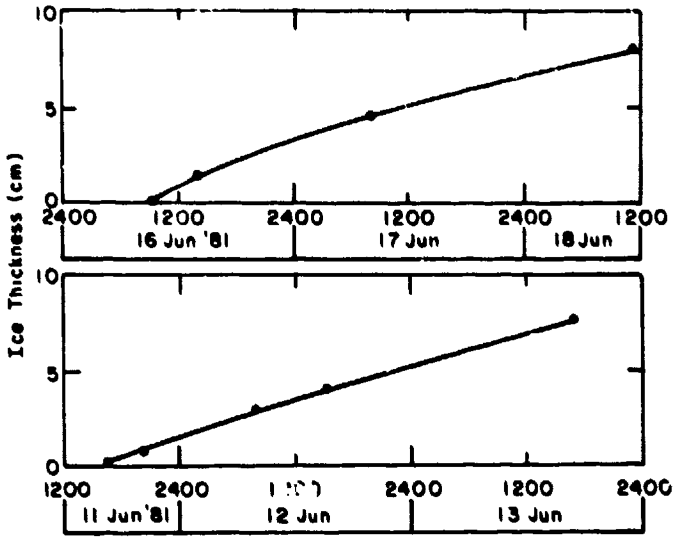


Figure 2. Typical ice growth curves for sheets grown in the CRREL tank. Average growth rates are 36–38 mm/day and are typical for ambient air temperatures of -20°C.

duced macrocystalline S1 ice dominated by crystals exhibiting vertical or near-vertical c-axes. In bottom ice (ice sheet growth in the tank was usually terminated after 9–12 cm of thickness was attained), the cross-sectional diameters of individual crystals in S1 type ice often exceeded the lateral dimensions of the thin sections, which measured 10 by 10 cm. As with seeded ice sheets, the thermal regime of the water column seemed to exert no significant influence on the texture or orientation of crystals in S1 type macrocrystalline ice. Full details of these and other factors affecting orientation textures in quietly frozen water are reported in Gow (1986).

As demonstrated in Figure 1, the highly contrasted structural characteristics of S1 and S2 ice grown in the test tank corresponded very closely with those observed in congelation lake ice. In several of the ice sheets, structure was examined at a number of widely spaced locations, mainly to determine if a particular orientation texture was being maintained over the entire ice sheet. No significant deviations in structure were observed, indicating that the tank was large enough to promote growth of ice sheets that were substantially free of edge effects. This is important when considering use of such a tank to grow uniformly textured ice sheets for mechanical properties testing. Another feature of ice grown in the tank was the general absence of air bubbles. Apparently, the rate of freezing was sufficiently slow (30–40 mm/day) to ensure rejection of virtually all dissolved air at the ice/water interface. The lack of air bubbles is also reflected in density measurements. These rarely yielded values less than 0.913 Mg/m^3, equivalent to porosities of less than 0.5%. Representative ice growth curves are presented in Figure 2.

Our successful fabrication of S2 and S1 ice in the tank at CRREL—by the simple expedient of seeding or not seeding the water prior to freezing—gave us added confidence in using these ice sheets as realistic analogues of congelation ice for mechanical properties testing. This testing, involving measurements of flexural strength and strain modulus of small beams as a function of both the temperature and orientation texture of the ice, was begun in February 1983 and completed in November of the same year. Preliminary results of these measurements are presented in Gow and Ueda (1984).

GROWTH CHARACTERISTICS OF EXPERIMENTAL ICE SHEETS

Both S1 (unseeded macrocrystalline) and S2 (seeded columnar) type ice sheets were investigated in the current series of small beam tests. Eight ice sheets (five seeded, two unseeded and one composed of seeded and unseeded portions) were grown in the tank. Three of these ice sheets were dedicated to the investigation of stress concentra-

Figure 3. Vertical and horizontal thin sections of seeded columnar S2 ice structure at different stages of growth in ice sheet 1, photographed between crossed polaroids to delineate the outlines of individual crystals. Scale subdivisions in photographs of horizontal thin sections of this and subsequent structure figures measure 1 mm. Note the very substantial increase in size of crystals between the top and the bottom of the ice sheet, a very characteristic feature of columnar ice growth.

tion effects at the roots of cantilever beams. Brief descriptions of the growth characteristics and structure of the individual ice sheets, together with some pertinent remarks concerning the beam tests themselves, are given below.

Ice sheet 1

This ice sheet was seeded on 22 February 1983, with the water in the tank cooled isothermally to 1.5°C. During the period of 23–28 February, the air temperature above the tank (initially set at -20°C to promote rapid initial freezing) was moderated to slow down ice growth. This resulted in no change in the texture or orientation of crystals in the ice sheet, which remained thoroughly transparent (bubble-free) throughout its entire thickness. Beam testing was begun on 28 February, on columnar-textured ice approximately 11 cm thick, and was completed on 7 March. The total number of beams tested was 38. Results of flexural strength and strain modulus measurements are fully tabulated in Appendix A. Examples of the crystalline structure of this S2 type ice sheet are shown in Figure 3.

Ice sheet 2

Water in the tank was cooled isothermally to 1.7°C prior to seeding on 25 March 1983. Testing was begun on 28 March and completed on 4 April, by which time a total of 128 beams had been tested. Vertical and horizontal structure sections of bubble-free S2 type ice characterizing this particular ice sheet are presented in Figure 4. Results of beam measurements are fully tabulated in Appen-

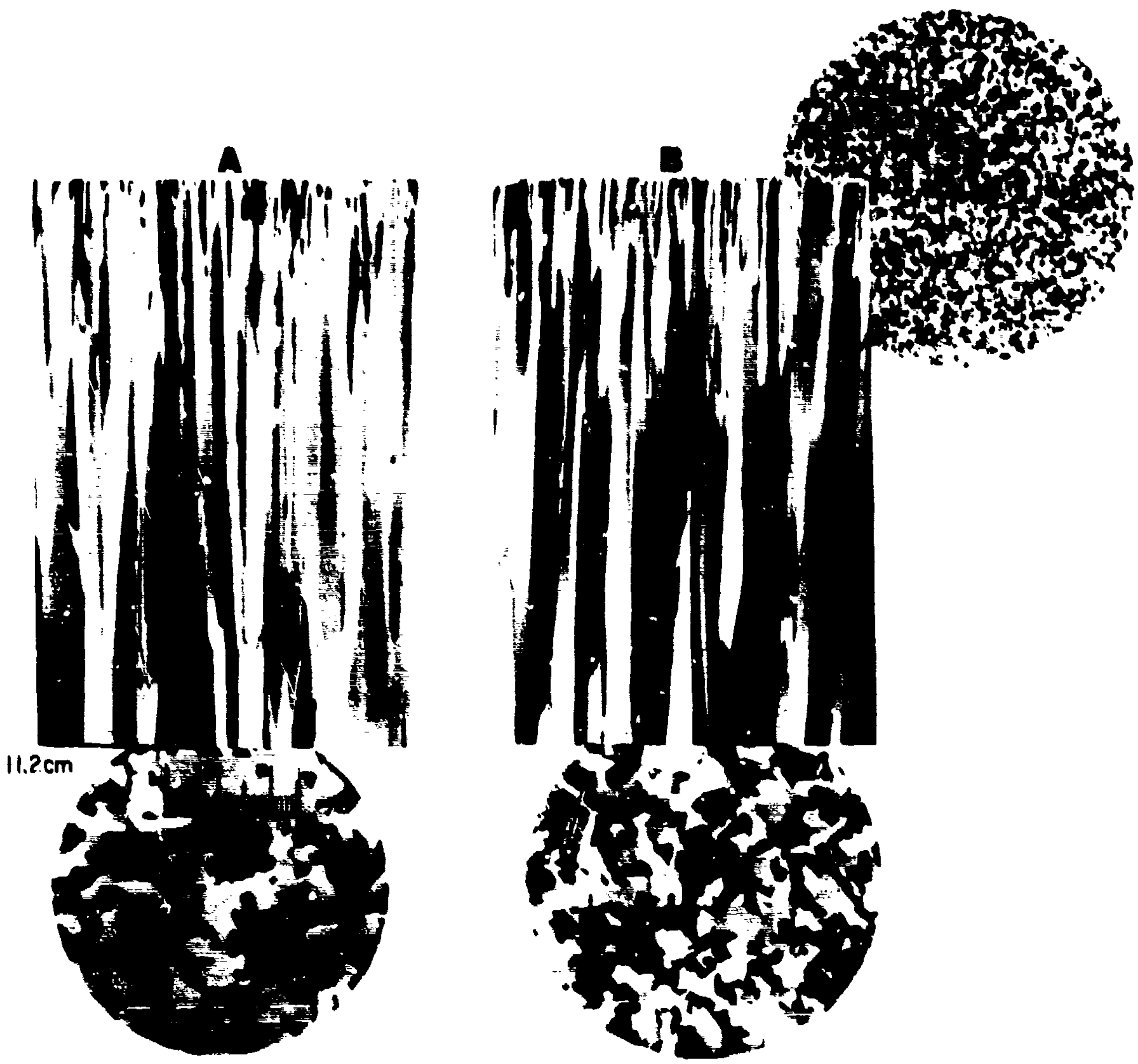


Figure 4. Vertical and horizontal thin sections of seeded (S2) columnar ice from two different locations on ice sheet 2. The crystal structures of both are essentially identical.

dix A. During testing we noted that simply supported beams tested in parallel with cantilever beams tended to yield off-center breaks, especially at the lowest ambient test temperatures. This behavior appears to be related to a temperature gradient effect since beams allowed to equilibrate to the ambient air temperature before testing (isothermal beams) only occasionally exhibited off-center failures.

Ice sheet 3

In this instance the water (previously cooled isothermally to 2.0°C) was allowed to nucleate spontaneously without seeding. Ice growth was initiated on 6 April 1983 and yielded an S1 type, bubble-free, macrocrystalline ice sheet. Actual testing of beams began on 9 April and was terminated on 14 April. This series of tests demonstrated, apparently for the first time, that S1 type ice was appreciably stronger in the cantilever mode than S2 type ice for both top and bottom in tension tests. Such a difference in behavior between the two types is attributed to a change in failure mechanism, linked to the existence of large crystals having vertical c-axes in S1 ice, where the failure plane, on the order of 100 cm^2 in most tests, may intersect only one or two crystals. In S2 ice the vertical failure plane rarely intersected *less* than 20 crystals. Total number of beams tested was 143. Appendix A contains a full tabulation of data. Representative structure sections are presented in Figure 5.

Ice sheet 4

This ice sheet was seeded on 22 April 1983 after

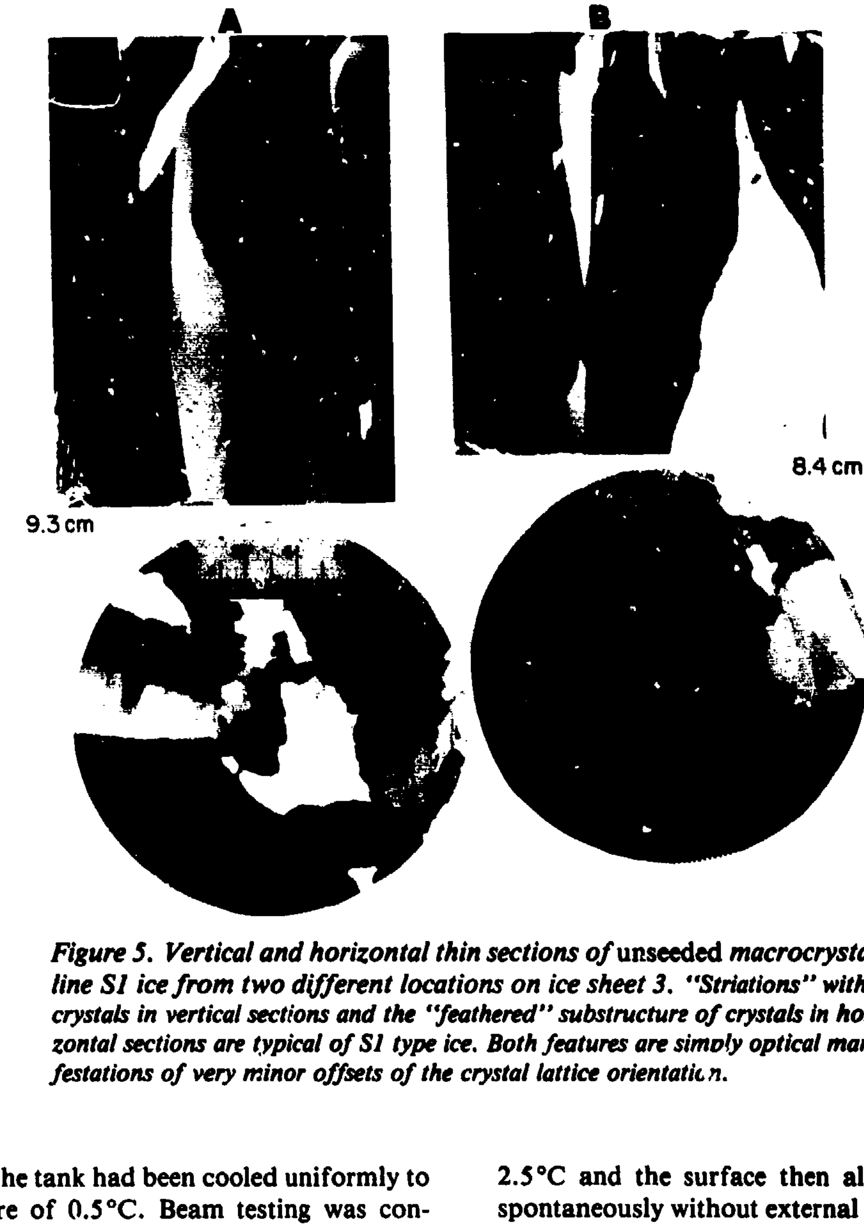


Figure 5. Vertical and horizontal thin sections of unseeded *macrocrystalline S1 ice from two different locations on ice sheet 3. "Striations" within crystals in vertical sections and the "feathered" substructure of crystals in horizontal sections are typical of S1 type ice. Both features are simply optical manifestations of very minor offsets of the crystal lattice orientation.*

the water in the tank had been cooled uniformly to a temperature of 0.5°C. Beam testing was conducted during the period 25 April–5 May. Investigations of this bubble-free ice sheet included experiments with changing the beam dimensions, including increasing the width by 60% with respect to the thickness, and with varying the length-to-thickness ratio from 7:1 to 10:1. No significant changes in the flexural strength of the ice were observed as a result of either of these changes in beam dimensions. The total number of beams tested was 114. Vertical and horizontal structure sections from two different parts of the ice sheet are shown in Figure 6. Data sets for the several batteries of tests are tabulated in Appendix A.

Ice sheet 5

The tank water was cooled isothermally to 2.5°C and the surface then allowed to nucleate spontaneously without external seeding on 13 May 1983. Beam testing was begun on 18 May and completed by 23 May. The total number of beams tested was 115 and data on the dimensions, flexural stength and strain modulus are included in Appendix A. Structure sections demonstrating the macrocrystalline, bubble-free nature of this S1 type ice sheet are shown in Figure 7.

Ice sheet 6

This was a two-part sheet consisting of seeded and unseeded portions. On 3 June 1983, after circulating the water to an isothermal temperature of 0.8°C, we covered half of the tank with a plastic sheet while the other half was seeded to initiate growth of S2, columnar-textured ice. As soon as seeding was completed the plastic sheet was re-

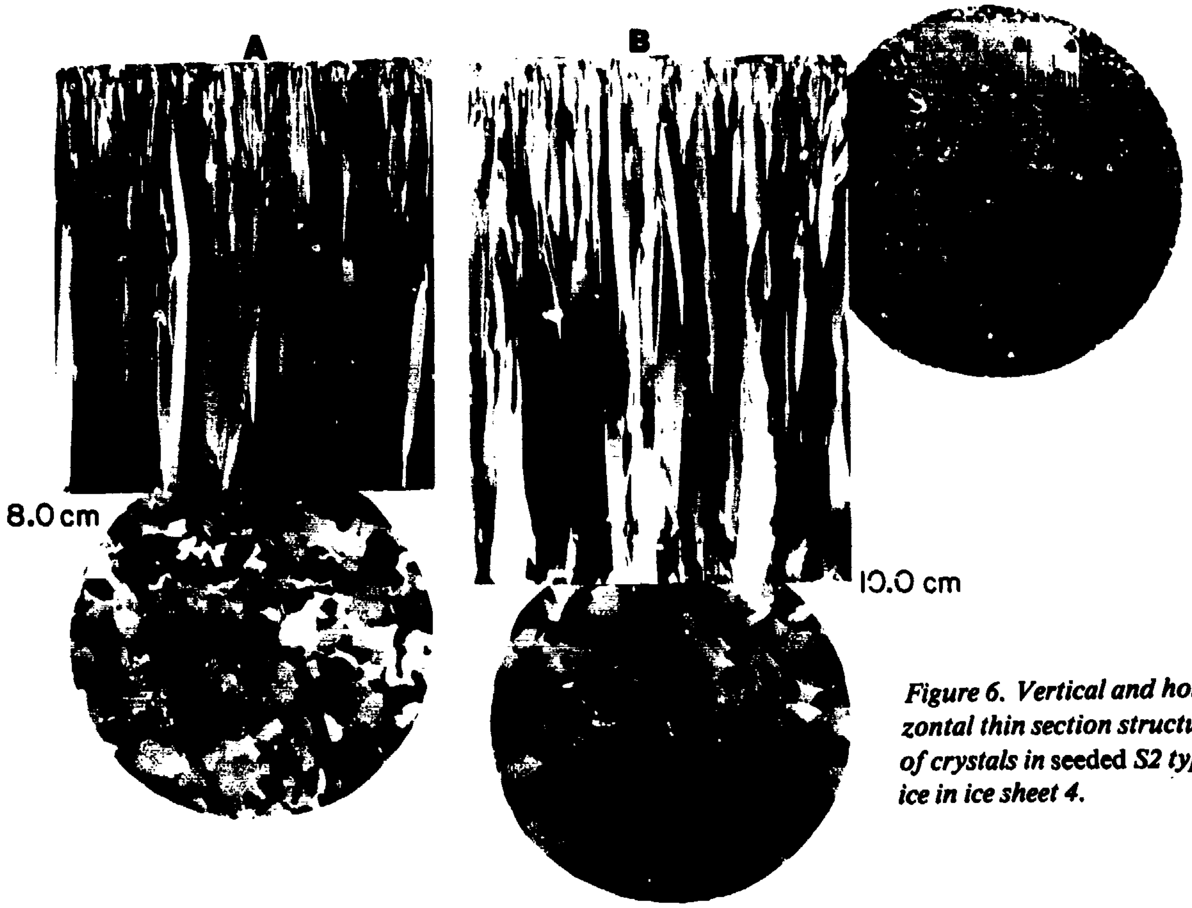


Figure 6. Vertical and hori zontal thin section structure of crystals in seeded *S2 type ice in ice sheet 4.*

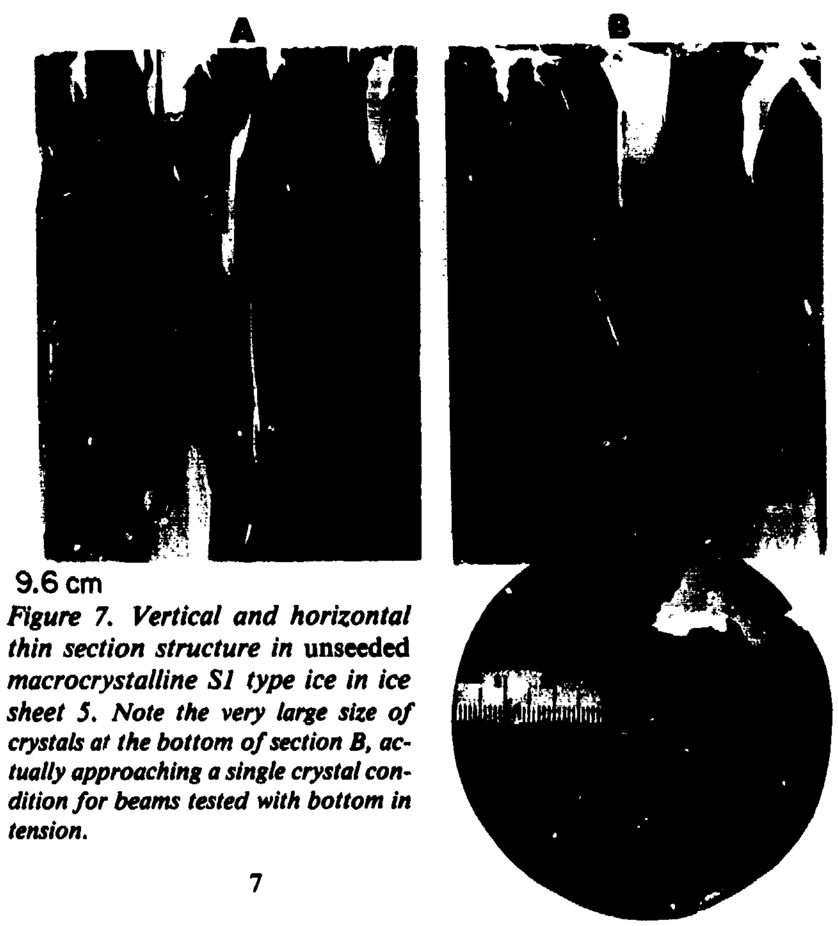


Figure 7. Vertical and horizontal thin section structure in unseeded *macrocrystalline S1 type ice in ice sheet 5. Note the very large size of crystals at the bottom of section B, actually approaching a single crystal condition for beams tested with bottom in tension.*

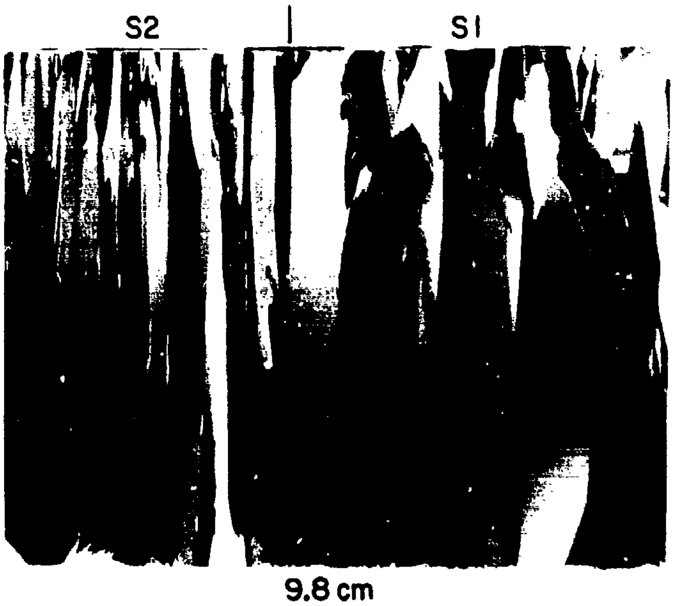


Figure 8. Vertical thin section from ice sheet 6 showing the transition between seeded *(S2) and* unseeded *(S1) ice types.*

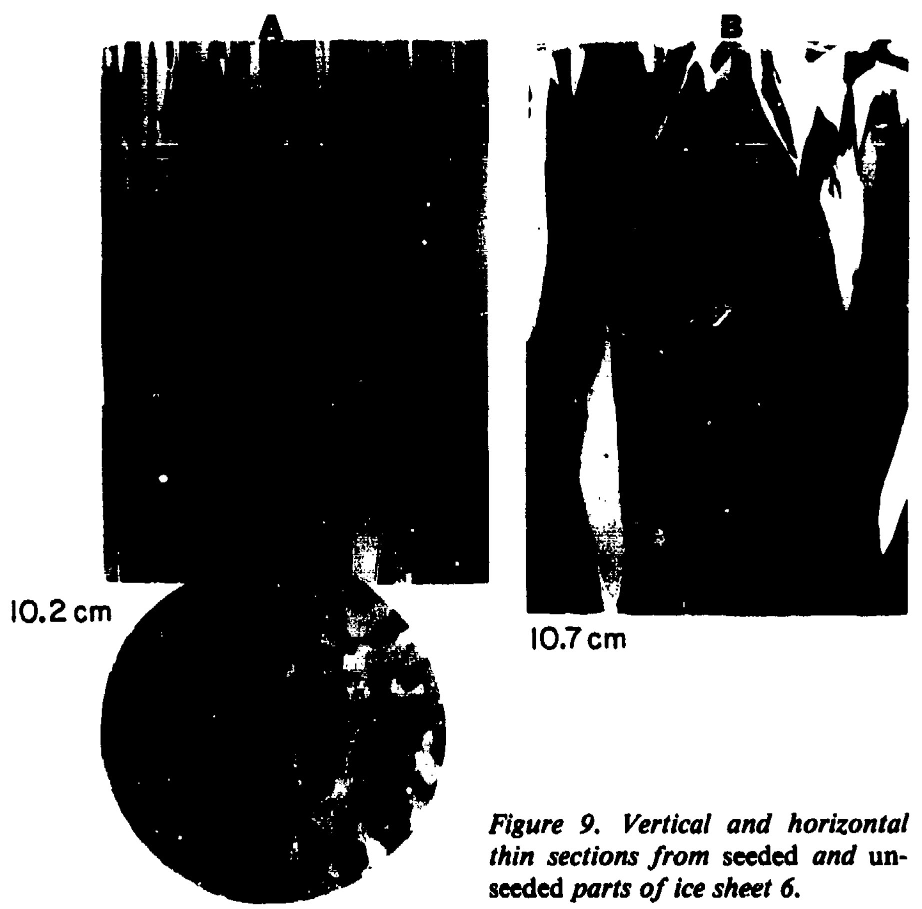


Figure 9. Vertical and horizontal thin sections from seeded *and* unseeded *parts of ice sheet 6.*

moved and the unseeded water allowed to nucleate spontaneously. This composite bubble-free ice sheet was the first of three ice sheets to be used to evaluate stress concentration effects at the roots of sharp-cornered cantilever beams. These tests involved the drilling of 10- and 20-cm-diameter stress relief holes at the fixed ends of the beams. Tests were begun on 6 June and completed 9 June. A total of 53 beams was tested. Detailed data for all tests are presented in Appendix A. A vertical structure section from the S1–S2 transition region is shown in Figure 8. Representative sections of S1 and S2 type ice are presented in Figure 9.

Ice sheet 7

A seeded ice sheet was produced from water cooled uniformly to 3.0°C prior to freezing on 8 July 1983. Beginning 11 July, further tests were made on stress riser effects at the fixed ends of cantilever beams. A total of 61 beams was tested before tests were concluded on 14 July. Full data sets are included in Appendix A. Typical examples of the columnar-textured, bubble-free structure of this ice sheet are shown in Figure 10.

Ice sheet 8

Water in the tank was cooled uniformly to 2.2°C prior to seeding on 4 November 1983. Tests were again focused on evaluating stress concentration effects at the fixed corners of cantilever beams: 78 viable beam tests were conducted during the period 7 to 9 November. Full data sets are given in Appendix A. Representative thin section photographs of crystal structure in this bubble-free ice sheet are shown in Figure 11.

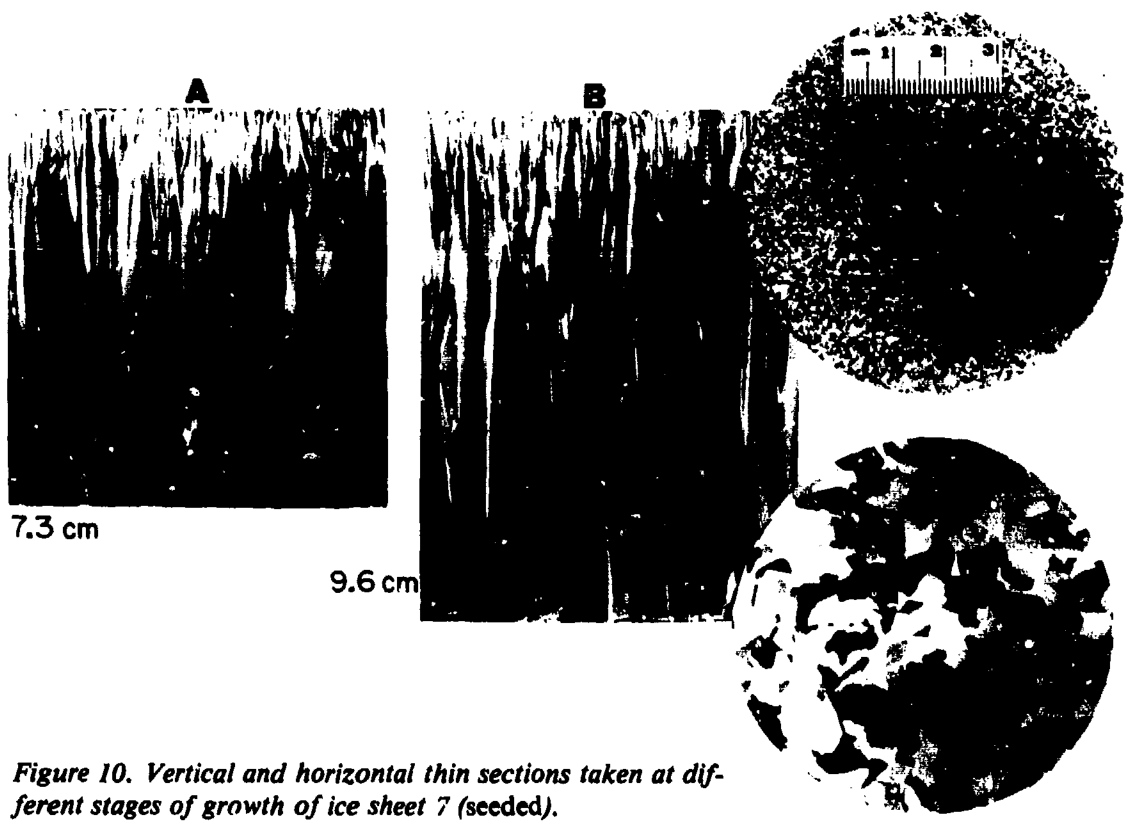


*Figure 10. Vertical and horizontal thin sections taken at different stages of growth of ice sheet 7 (*seeded*).*

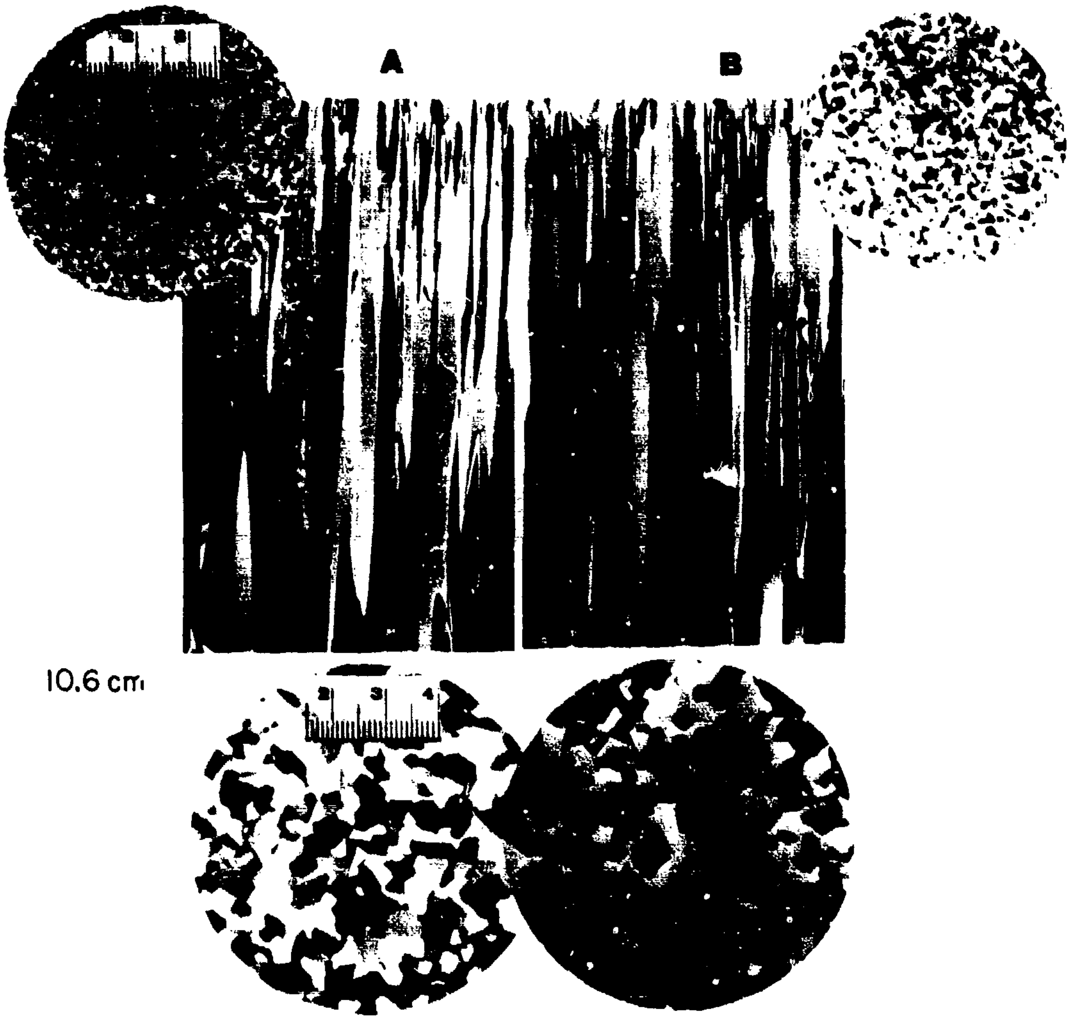


Figure 11. Vertical and horizontal thin sections of seeded columnar S2 type ice from two different locations on ice sheet 8.

NATURE OF TESTING PROGRAM

Testing entailed measurements, initially, on cantilever beams that were divided into two groups: those tested in parallel with the cantilever beams (these measurements were performed in three-point loading immediately following the cantilever tests to ensure that temperature profiles in the ice remained the same for both kinds of beam tests), and those beams that were allowed to equilibrate to the ambient air temperature to facilitate testing of isothermal beams. Measurements were conducted at ambient air temperatures of -1, -5, -10 and -19°C on a total of 730 beams.* These included 312 cantilever tests, 166 beams tested in the simple support mode in parallel with the cantiliver beams and simple support tests of 252 isothermal beams.

The intrinsic value of cantilever beam tests is that they are carried out in situ and are relatively easy to do. Because such tests take account of any effect of temperature gradients* in an ice sheet, they also furnish direct measurements of the flexural strength of the ice, provided due considera-

*The number of beams actually prepared exceeded 800 but because of accidental breakage, instrument malfunctions, etc., useful data were obtained on 730 beams only.

*In in-situ tests of this kind, the ultimate strength of the ice sheet must be related in some degree to temperature gradients resulting from differences in temperature between the top of the ice sheet and the bottom, which must necessarily be at 0°C. For example, at an ambient air temperature of -19°C in ice 10-12 cm thick, the temperature gradient effect should be a significant factor in determining the precise manner of tensile failure, and hence, the flexural strength of the ice.

tion is given to possible stress concentration effects at the fixed ends of cantilever beams. Testing the same beams in the simple support mode should suppress any stress riser effects. Accordingly, differences in flexural strength between cantilever beams and the same beams tested in the simply supported mode should at least indicate the extent of the stress concentration effect at the root of the cantilever beams, a major consideration of results presented in this report. The main purpose of measuring the flexural strengths of isothermal beams of S1 and S2 ice was to evaluate the effects of grain size and crystal orientation changes as a function of ice temperature. Additionally, results of these tests on isothermal beams and those obtained on simply supported beams, tested in parallel with cantilever beams, were used to assess temperature gradient effects.

EXPERIMENTAL TECHNIQUES

Beam preparation

After scribing the desired beam arrangement on the ice surface with a chisel, we used a small electric circular saw, capable of cutting ice 12 cm thick, to prepare beams with straight parallel and vertical sides (Fig. 12). In practice the circular saw was used to cut to a depth of about three quarters of the ice thickness, a coarse-toothed timber saw then being used to cut the remaining quarter.

Cantilever beam breaker

The cantilever beam breaker (Fig. 13) consisted basically of a manually operated screwjack with a threaded rod of 1.6 mm (0.063 in.) pitch pushing on a spring-loaded plunger. The plunger was fitted with an Interface Model SM-100, 445-N (100-lb) capacity load cell, to which a C-shaped member was attached and loosely clamped to the free end of the cantilever beam. With this device cantilever beams could be tested in either the pull-up (bottom in tension) or push-down (top in tension) modes. The screwjack assembly was attached to a frame that could be clamped firmly to one of a pair of heavy 31-cm (12-in.) I-beams spanning the center of the tank. The tank was large enough to allow about 40 beams to be cut and tested on each side of the I-beams. Temperatures at the tops of beams were measured with either dial stem thermometers accurate to -0.5°C, or mercury thermometers with a measurement precision of -0.2°C. Bottom ice, naturally, remained at 0°C. A Schaevitz LVDT (Linear Variable Differential Transformer) with a sensitivity of 3.2 V/mm was used to measure beam tip deflections. The measur-

Figure 12. Technique used to prepare ice beams with straight vertical and parallel sides. In this instance beams are being prepared for in-situ cantilever testing, followed by testing in the simple support mode.

Figure 13. Experimental setup for measuring flexural strengths and strain moduli of cantilever beams. This apparatus, clamped firmly to the I-beam spanning the tank, permits testing in both the push-down and pull-up modes.

Figure 14. Closeup of beam deflection measuring device. Device is moved to the free end of the beam immediately before testing. Deflections are measured relative to the uncut ice adjoining the cantilever beam.

ing device is shown being positioned in Figure 14. Measurements were made relative to the uncut, adjoining ice.

The design of the beam breaker permitted the rate of beam loading to be controlled readily by the operator cranking the handle of the screw-jack. The majority of beams were loaded to failure in less than 1 second (the time from load take-up to failure). Signals from the load cell and LVDT were transmitted to a Vishay BA-4 signal conditioner and recorded versus time on a two-channel Gould-Brush 222 strip chart recorder. A typical example of data output is shown in Figure 15.

Cantilever beam lengths averaged 105–110 cm and the ratio of length to width to thickness averaged 10:1:1. Measurements in which this ratio was changed to 7:1:1 in one battery of tests, and to 10:1.6:1 in another, yielded no significant changes in calculated values of either the flexural strength or strain modulus.

Flexural strength, S, and strain modulus, E, were calculated from simple elastic beam theory using the equations:

$$S = \frac{6PL}{wh^2} \tag{1}$$

and

$$E = \frac{4}{w}\left(\frac{L}{h}\right)^3 \frac{P}{d} \tag{2}$$

where P = failure load
L = length of the beam from point of failure to the point of load application
w = width measured at the failure plane
h = thickness measured at the failure plane
d = beam tip deflection at failure.

The calculated values of flexural strength and strain modulus are estimated to be accurate to

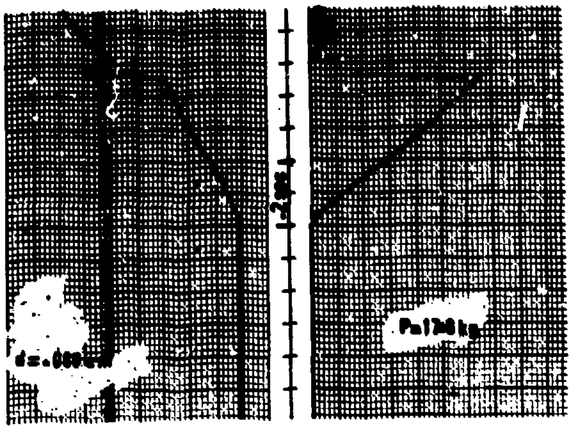


Figure 15. Load-time and deflection-time records for cantilever beam 21 from ice sheet 2 tested with bottom in tension. Time to failure was 0.45 seconds.

±4%. However, these values are necessarily subject to certain assumptions implicit in the formulation of eq 1 and 2. These assumptions, particularly those concerning the homogeneity and isotropic condition of the material being tested, are rarely satisfied in either natural or laboratory-grown ice sheets. Indeed, a major aim of the present work was to assess the effect, on flexural characteristics, of departures from an isotropic, homogeneous medium—for instance, grain size variations and crystal orientation changes in the ice. Since the measurement techniques closely followed the guidelines recommended by Schwarz et al. (1981) for small beam testing per se, the data obtained in the current series of tests are considered to represent reasonable index values of the flexural characteristics of freshwater model congelation ice sheets grown in the CRREL tank.

Breaker for simply supported beams

This device (Fig. 16) consisted of an I-beam main frame with two cylindrical reaction bars that could accommodate beams between 71 and 102 cm long. A three-point* loading arrangement was used in which force was applied to the center of the beam by means of a manually operated wormgear screwjack, having a 4450-N (1000-lb) capacity, and attached to the midpoint of the main

*Four-point loading is generally advocated on the assumption that such an arrangement eliminates the shear stresses in the length of beam between the applied loads, and that the maximum moment occurs along the length of beam between the applied loads and not at a single point. However, Timco and Frederking (1982), in tests on similar freshwater ice sheets, observed no significant differences in strength between three-and four-point loading arrangements.

Figure 16. Breaker for simply supported beams, with deflection device in place, in readiness for test. Breaker is designed for three-point loading and can accommodate beams 71 to 102 cm long with width and thickness dimensions of up to 14 cm.

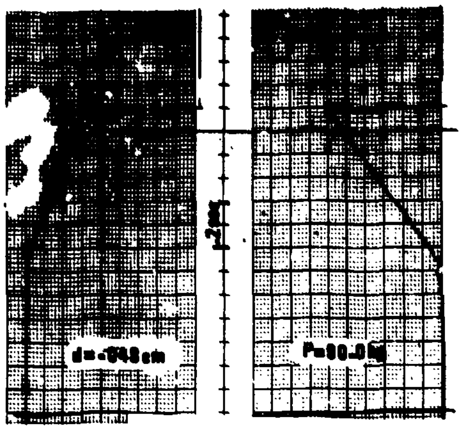


Figure 17. Load-time and deflection-time records for simply supported beam 8 from ice sheet 3 tested with top in tension. Time to failure was 0.7 seconds.

frame, which itself was clamped firmly to a 31-cm I-beam spanning the center of the tank. Force was distributed across the width of the beam through a transverse bar. A 2230-N (500-lb) capacity Interface Model SM-500 load cell located between the screwjack and distribution bar was used to sense the load. One turn of the screwjack provided 1.27 mm of vertical displacement. With this setup, time to failure from initial takeup of the load required less than 1 second. Center deflections were measured with the same LVDT that we used for the cantilever beams. The LVDT was attached to a bar, supported by two legs resting on the beam directly above each reaction point; it was located slightly to the side of the transverse bar so as not to interfere with the loading mechanism.

As with the cantilever tests, the load cell and transducer signals were transmitted to a two-channel strip chart recorder. A typical example of data output from a three-point loading test is shown in Figure 17. The length-to-width-to-thickness ratio of simply supported beams averaged 9:1:1. The flexural characteristics of beams were calculated on the basis of

$$S = \frac{3}{2}\frac{PL}{wh^2} \tag{3}$$

and

$$E = \frac{1}{4w}\left(\frac{L}{h}\right)^3\frac{P}{d} \tag{4}$$

where terms in these equations are the same as those for eq 1 and 2. In the case of simply supported beams, L is the distance between the two end supports (beam span) and d is the mid-point deflection at failure. The weight of the beam was also taken into account when we calculated its flexural strength and strain modulus, the values of which are estimated to be accurate to ±5%. Periodic measurements of temperatures at the tops and bottoms of beams were made with mercury thermometers having a measurement precision of ±0.2°C.

RESULTS AND DISCUSSION

Cantilever beams

Detailed results for all beams tested in the current series of measurements are included in Appendix A. Averaged values for individual ice sheets are listed in Table 1 (flexural strength) and Table 2 (strain modulus). Weighted averages of flexural strength and strain modulus for both S1 and S2 ice sheets at the four test ambient air temperatures are also included in Tables 1 and 2 and these data are plotted in Figures 18 and 19 respectively.

Conventional cantilever beam tests

Results of conventional cantilever beam tests on S2 ice (Fig. 18) show only a weak dependence of strength on surface temperatures for beams tested with the top in tension, flexural strengths increasing from about 700 kPa at −1°C to only about 900

Figure 18. Variation with temperature of the flexural strength of cantilever beams of S1 and S2 ice. Symbols T and B refer to top and bottom in tension tests respectively. Note that in bottom in tension tests, temperatures of the fiber in tension are necessarily at 0°C. Number of beams used to determine average flexural strength values for each data point are also indicated.

Table 1. Average flexural strengths (kPa) of cantilever beams.

Ambient temperature (°C)	Seeded ice sheets No. 1	No. 2	No. 4	No. 7	No. 8	Unseeded ice sheets No. 3	No. 5	No. 6	Average Seeded	Average Unseeded
- 1		622*	754		711	958	997		687 ± 81	975 ± 132
		689†	789		743	1040	1053		741 ± 84	1045 ± 147
					686					
					754					
- 5	714	673		904		888	907	871	735 ± 121	891 ± 104
	—	705		812		1165	1297	954	743 ± 67	1181 ± 183
	724	778								
	—	717								
-10		734	766			824	1133		786 ± 89	943 ± 183
		739	815			1188	1212		767 ± 75	1199 ± 106
			905							
			745							
-19		859	946			922	940		903 ± 107	930 ± 237
		680	874			1164	1062		745 ± 134	1123 ± 167
Total beams	12	67	40	7	19	60	40	4	145	104

* Top in tension.
† Bottom in tension.

Table 2. Average strain modulus (GPa) of cantilever beams.

Ambient temperature (°C)	Seeded ice sheets No. 1	No. 2	No. 4	No. 7	No. 8	Unseeded ice sheets No. 3	No. 5	No. 6	Average Seeded	Average Unseeded
- 1			4.4*		5.2	5.4	5.8		4.9 ± 0.6	5.6 ± 0.9
			4.1†		4.6	5.7	5.6		4.4 ± 0.4	5.7 ± 0.4
- 5	4.[illegible]	4.4				6.6	6.1	4.6	4.3 ± 0.7	6.3 ± 0.9
	—	4.7				6.8	5.8	5.6	4.7 ± 0.9	6.3 ± 0.9
-10			4.6							
			4.9							
		5.4	4.5			6.8	6.0		4.7 ± 0.9	6.4 ± 1.2
		5.0	4.5			6.1	5.1		4.6 ± 0.4	5.7 ± 0.8
-19		—	4.9			5.4	6.0		4.9 ± 0.4	5.6 ± 0.9
		5.0	4.9			5.7	5.0		4.9 ± 0.4	5.4 ± 1.2
Total beams	11	31	38		19	56	37	4	99	97

* Top in tension.
† Bottom in tension.

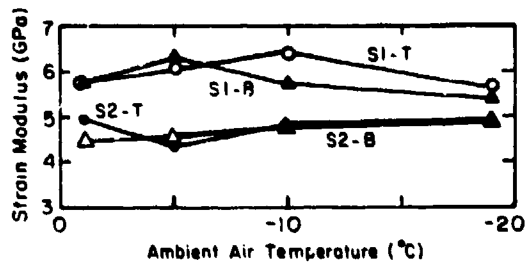


Figure 19. Strain modulus data of cantilever beams of S1 and S2 ice versus temperature measured at the top of the ice sheet. Symbols T and B refer to top and bottom in tension tests respectively.

kPa at -19°C. For beams tested with their bottoms in tension (tension fiber at 0°C), flexural stength showed even less dependence on temperature, strengths remaining remarkably constant at around 750 kPa over the same range of ambient air temperatures. Part of the increased strength shown by beams tested with the top in tension may reflect the effect of the smaller grain size in the tops of S2 (columnar-textured) ice sheets with decreasing test temperatures. Beams of S2 ice characteristically failed in vertical planar fashion. When viewed in reflected light, the individual outlines of columnar crystals could be clearly delineated on the fracture surface.

Timco and Frederking (1982) measured an average value of 770 kPa for S2 ice beams tested with top in tension at -10°C. This compares very closely with the average value of 786 kPa we obtained on 17 beams from two S2 type ice sheets tested at the same temperature. Lavrov (1971) reports somewhat higher values for his "structurally simulated ice" (S2 ice equivalent), on the order of 1100 kPa for beams tested at -7 to -5°C with the top in tension. Neither Timco and Frederking nor Lavrov tested beams with bottoms in tension, nor did they examine the flexural strength of macrocrystalline S1 ice sheets.

Tests on S1 ice (Fig. 18) also failed to show any really systematic change in strength with changing surface air temperature, for either top or bottom in tension tests. However, S1 ice tested stronger overall than S2 ice. Push-down tests (top in tension) averaged around 950 kPa, whereas those beams tested in the pull-up mode (bottom in tension) ranged in strength from 1000 to 1200 kPa. This increased strength of S1 type ice (approximately 30–40% greater than that of S2 ice) reflects both the effects of the near-perfect vertical c-axis alignments of crystals in S1 ice and the very large sizes of crystals themselves. In bottom in tension tests, for example, tensile failure frequently involved just one or two crystals in the beam cross sections. This approach to single crystal failure, which often promoted conchoidal fracture surfaces, more than compensated for the fact that the bottom ice was at or close to 0°C. The net result is that S1 ice with bottom in tension tested the strongest of all cantilever beams. This obvious control of oriented crystal structure in enhancing the flexural strength of S1 ice, together with the weak to virtual non-dependence of strength of both S1 and S2 ice on ambient air temperatures over the range -1 to -19°C, are the most striking features of the cantilever beam tests conducted in the CRREL tank.

In these and other tests of laboratory-grown ice sheets, the flexural strengths of cantilever beams are generally much higher than those measured in the field. For example, the maximum strengths measured by Gow et al. (1978) on large cantilever beams of lake ice never exceeded 1000 kPa, and these were only observed in the coldest ice that was composed of snow ice with grain diameters on the order of 1 mm. Additionally, Gow et al. (1978) reported a significant decrease in the flexural strength of the ice with increased exposure to elevated air temperatures and solar radiation during the late winter and spring. This exposure leads to a degrading of the ice structure that in the extreme case manifests itself in the form of grain boundary melting and candling. This is accompanied by significant loss of strength to values of 400 kPa or less. However, such behavior was not observed in any of the ice sheets in the CRREL test tank, not even in those sheets held at temperatures of 0°C for extended periods of time nor in beams removed from the water and also held at 0°C for long periods. Nor did the ice lose structural integrity—there was no sign of crystal boundary modification or candling, for example. Such observations strongly suggest that solar radiation (not a factor in an indoor tank) is a major influence in promoting candling and concomitant loss of flexural stength in natural ice covers. This point was subsequently demonstrated when blocks of ice were taken from the tank and found to undergo rapid candling when exposed to sunlight at air temperatures around 0°C.

Our conventional cantilever beam tests also included a series in which changes in the dimensions of beams of S2 ice were investigated to determine the effect, if any, of such changes on the flexural properties of the ice. The tests were similar to those performed by Frederking and Timco (1983) who reported that the flexural strength of S2 ice is essentially independent of length but decreases with increasing beam width. The results of beam length change, based on tests from ice sheet 4 (28 April test series, Appendix A), tabulated below, show a slight but not statistically significant change in strength for a $L{:}w{:}h$ change from 11:1:1 to 8:1:1. Data are in accord with those of Frederking and Timco (1983).

$L{:}w{:}h$	$L{:}w{:}h$
11:1:1	8:1:1
$\bar{S}_T = 754$ kPa	$\bar{S}_T = 777$ kPa
$\bar{S}_B = 796$ kPa	$\bar{S}_B = 833$ kPa

where B indicates bottom in tension and T top in tension. Also, measurements involving a 60% increase in beam width, while keeping the length and thickness constant (25 April test series, Appendix A), resulted in no significant change in flexural strength of the beams.

$L{:}w{:}h$	$L{:}w{:}h$
10:1:1	8:1.6:1
$\bar{S}_T = 766$ kPa	$\bar{S}_T = 741$ kPa
$\bar{S}_B = 770$ kPa	$\bar{S}_B = 807$ kPa.

This result might seem at variance with the reported conclusion of Frederking and Timco (1983) that increasing the beam width decreases the strength. However, an inspection of the data in Figure 8 of their paper shows that for beam width changes of between one and two times the beam thickness, flexural strength actually increased (from about 750 kPa to nearly 1000 kPa) before decreasing progressively to about 500 kPa at beam widths four times the thickness

These observations that flexural strength is not significantly influenced by beam width changes of between one and two times the beam thickness help resolve a difference in guidelines for small beam testing recommended by Schwarz et al. (1981) and Lavrov (1971). Whereas Schwarz et al. recommended that beam widths should measure one to two times the beam thickness, Lavrov advocated the use of beams with a square cross section. Both recommendations appear valid. In most of the tests reported here, beams with a square cross section were used.

Individual measurements of strain modulus are listed in Appendix A and averaged values for cantilever beams are presented in Table 2. Averaged values of strain modulus based on all tests at the four test temperatures are plotted in Figure 19. S2 ice showed no significant differences in modulus between top or bottom in tension tests at any temperature, and no significant trend with temperature per se was observed either. Results for S2 ice then are that strain moduli, ranging between 4 and 5 GPa, are essentially independent of temperature over the range −1 to −19°C. S1 ice beams similarly showed no significant differences in modulus between top and bottom in tension tests and values again appear virtually independent of temperature. However, strain moduli of S1 ice are appreciably higher than those of S2 ice, 5 to 6 GPa or on the order 10 to 20% larger. Lavrov (1971) reported strain moduli of about 2 GPa for beam strengths of 1000–1100 kPa in S2 ice. Timco and Frederking (1982) reported strain moduli of 1.6 GPa for top in tension tests of S2 ice with flexural strengths of about 770 kPa. However, Timco and Frederking, unlike Lavrov (1971), found no dependence of strain modulus values on loading rate. The most recent data are from Frederking and Svec (1985) who, while measuring flexural characteristics of freshwater ice in an outdoor pool, obtained strain modulus values of 5.4 GPa for fine-grained ice at the top of the ice sheet. These data are similar to ours (4 to 5 GPa) that were obained on fine-grained congelation ice at the tops of S2 ice sheets. Variations between the different observers probably reflect differences in both test techniques and ice types. Lavrov (1971), for example, appears to have incorporated results of tests from both laboratory-grown and natural ice covers.

Modified cantilever beam tests

In addition to testing conventional cantilever beams, we also dedicated parts of three ice sheets, numbers 6, 7 and 8, to studies of stress concentration effects at the roots of modified cantilever beams. Evidence for the existence of stress concentrations has been obtained mainly from field testing of large cantilever beams, but opinions as to the magnitude of such an effect vary widely. Both Butyagin (1966) and Lavrov (1971) argue against the existence of significant external stress risers, Butyagin on the basis of comparative tests of cantilever and simply supported beams that failed to show any significant difference in strength between the two, and Lavrov on the basis of tests on cantilever beams with their root sections flared to reduce external stress concentrations. However, Lavrov observed that the flexural strengths of simply supported beams generally exceeded those of cantilever beams. This Lavrov attributed to fundamental differences in the mechanics of failure of simply supported and cantilever beams.

Gow et al. (1978) conducted tests on both cantilever and simply supported beams of temperate lake ice and found that the ratio of flexural strength of cantilever to simply supported beams varied from 1:1 at cantilever beam strengths of around 400 kPa to 1:2 for cantilever beam strengths of 900 kPa (the same beams tested in the simple support mode failing at about 1800 kPa). This behavior was attributed to the effect of stress concentrations at the sharp-cornered roots of cantilever beams, with the maximum effect occurring in cold ice substantially free of structural imperfections. This explanation implied that the magnitude of the stress concentration effect depends on both the thermal and structural condition of the ice, and that in ice that has undergone extensive thermal degradation, leading to loss of cohesion between the grains and crystals of ice, the stress riser effect may be eliminated altogether. Määttänen (1976), working with beams of brackish water ice, found that cantilever beams with a large radius of curvature at the root were about 30% stronger than sharp-cornered conventional beams. Gow and Ueda (1984), experimenting with freshwater model ice, also reported significant increases in flexural strength of cantilever beams when their roots were rounded out by drilling. Frederking and Svec (1985), conducting tests on 35-cm-thick ice in a large outdoor pool, also found that cantilever beams with holes drilled at the roots tested approximately 25% stronger than beams with roots terminated by parallel saw cuts.

To evaluate stress concentration effects in the current series of tests, measurements were made in which the normally sharp corners produced by parallel saw cuts at the roots of conventional cantilever beams were filleted by drilling 20-cm-diameter holes. We prepared these beams by first drilling 20-cm-diameter holes at intervals of 30 cm between centers and then making parallel saw cuts perpendicular to the holes so as to intersect adjacent drill holes tangentially. This arrangement, including the preparation of conventional cantilever beams alongside those with modified roots, is depicted in Figure 20. A total of 83 beams was tested, including 54 beams with filleted roots, 11 of which consisted of S1 ice and the remaining 43 of S2 ice. All beams of S1 ice and 28 of the 68 S2 ice beams were tested at an ambient air temperature of -5°C. The remaining 40 S2 ice beams were tested at an ambient air temperature of -1°C.

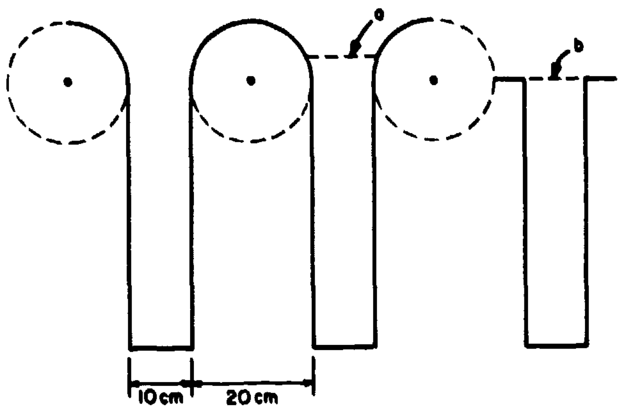


Figure 20. Layout of beams used to investigate stress concentration effects at the roots of cantilever beams. Arrows indicate usual locations at which failure occurred for (a) filleted and (b) conventional, sharp cornered beams.

Filleting the roots of beams invariably resulted in small to substantial increases in flexural strength, depending on the structure of the ice, its temperature and the particular surface in tension. Results (averaged values) are listed in Table 3. These tests in effect repeated earlier experiments by Lavrov (1971), but unlike Lavrov's results our filleted beams generally failed some distance back, occasionally as much as 5 cm into the region of

Table 3. Cantilever beam strengths (kPa); evaluation of stress concentrations at beam roots (number of tests in each battery is shown in parentheses).

Test temperature (°C)	T_u*	T_m	$\frac{T_m}{T_u}$	B_u	B_m	$\frac{B_m}{B_u}$
			Seeded (S2) ice			
-1	699 (9)	939 (11)	1.34	749 (10)	865 (11)	1.15
-5	904 (3)	1199 (12)	1.33	812 (4)	868 (9)	1.09
			Unseeded (S1) ice			
-5	871 (2)	1166 (3)	1.34	954 (2)	1229 (4)	1.29

* T_u = unmodified, top in tension; B_u = unmodified, bottom in tension; T_m = modified, top in tension; B_m = modified, bottom in tension.

Table 4. Cantilever beam moduli (GPa); tests of stress concentrations at beam roots.

Test temperature (°C)	Seeded (S2)				Unseeded (S1)			
	T_u*	T_m	B_u	B_m	T_u	T_m	B_u	B_m
-1	4.9	3.2	4.5	3.3	—	—	—	—
	5.5	3.8	4.8	3.4	—	—	—	4.2
-5	—	—	—	—	4.6	4.9	5.6	4.3

* T_u = unmodified, top in tension; B_u = unmodified, bottom in tension; T_m = modified, top in tension; B_m = modified, bottom in tension.

curvature (see cover), and at forces up to two times those needed to cause failure of conventional cantilever beams. The latter beams generally failed at or very close to the ends of the saw cuts; in both the modified and conventional beams the failure surface was vertical and planar.

Tests of S2 ice (see Table 3) show the same dependence of strength on temperature as demonstrated in Figure 18, with both modified and unmodified beams also testing strongest when the top surface was placed in tension. This probably reflects the effects of smaller grain size and lower temperatures at the top of the ice sheet. As a group, filleted beams of S2 ice, made to fail with their tops in tension, tested 30–35% stronger than unmodified beams. On the other hand, bottom in tension tests yielded much smaller differences in strength, filleted beams being on the order of 7–15% stronger. The results of our top in tension tests agree very closely with those obtained by Frederking and Svec (1985), who also found that introducing stress relief holes at the roots of cantilever beams increased flexural strength by 25–30% over that of conventional, sharp-cornered beams.

Tests of S1 ice beams also yielded increased strengths for filleted beams on the order of 30% for both top and bottom in tension, very similar to those of S2 ice beams tested with top in tension.

Modifying the roots of S2 ice beams also appears to exert some effect on the strain moduli, those beams with stress relief holes exhibiting lower values than conventional sharp-cornered cantilever beams (Table 4). No such effect was observed in beams of S1 ice. Results obtained on S2 ice beams with top in tension show the same trends as those found by Frederking and Svec (1985), who measured strain moduli of 5.4 GPa for unmodified beams compared to values of about 4.5 GPa for beams with stress relief holes drilled at the roots.

Parallel simply supported beams

This group of beams included all beams tested in parallel with cantilever beams. Tests of this kind involved transferring the cantilever beams from the water to the simply supported beam breaker. The actual tests were conducted in less than 2 minutes after we removed the beams from the water, thereby ensuring minimal changes in the thermal condition of the beams (ambient air temperature at the top of the beam, with the bottom at or very close to 0°C). A major reason for performing these tests in parallel with cantilever beams was to evaluate the effects of stress concentration at the sharp-cornered roots of cantilever beams; such effects should not exist in beams when both ends are freely supported. A second, but no less important, reason for carrying out parallel beam tests was to investigate the effects of temperature gradients in these ice beams via comparisons with *isothermal* beams of identical crystal structure tested in the same simple support mode.

Most (70%) of the simply supported beams in this series failed directly beneath the region of load application as transmitted through the transverse loading bar. The resultant fracture surfaces were generally vertical and planar, the only exceptions occurring with off-center breaks where fracture planes tended to be curved in the manner depicted in Lavrov (1971, p. 38). However, the percentage of off-center breaks tended to increase with decreasing ambient air temperature, indicating that differences in temperature between the tops and the bottoms of beams (temperature gradient factor) might influence the mechanism of failure and, possibly, the ultimate strength attained.

Detailed results of all beams tested in the parallel simple support mode are listed beside the corresponding cantilever beam data in Appendix A. Averaged values of flexural strength and strain modulus are presented in Tables 5 and 7 respec-

Table 5. Average flexural strengths (kPa) of parallel simply supported beams.

Ambient temperature (°C)	Seeded ice sheets No. 1	No. 2	No. 4	No. 7	No. 8	Unseeded ice sheets No. 3	No. 5	No. 6	Average Seeded	Unseeded
- 1		1469*	1529			1068	1566		1499 ± 85	1276 ± 271
		846†	1006			1066	1359		935 ± 117	1163 ± 197
- 5		1361				1215	1386		1586 ± 297	1281 ± 139
		—				1114	1393		1043 ± 147	1215 ± 268
		1810								
		1043								
-10		1521	1319			1182	1485		1454 ± 292	1292 ± 208
		899	991			1217	1685		922 ± 131	1412 ± 316
-19		1537	1788			1064	1462		1621 ± 356	1230 ± 242
		822	1052			1382	1329		673 ± 140	1364 ± 127
Total beams		46	21			58	35		67	93

* Top in tension.
† Bottom in tension.

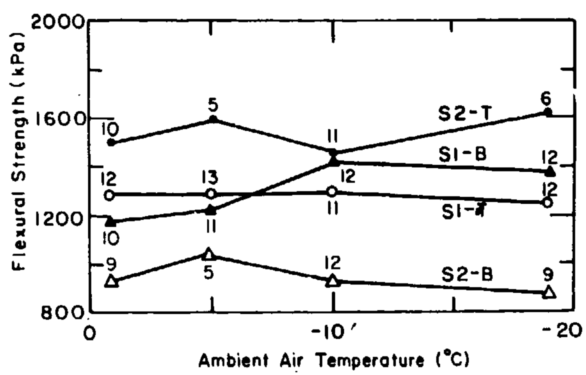


Figure 21. Variation with temperature of the flexural strength of simply supported beams of S1 and S2 ice tested in parallel with cantilever beams. Again, temperatures at bottoms of beams are at or very close to 0°C. Symbols T and B refer to top and bottom in tension tests respectively. Number of beams tested at each temperature is also indicated.

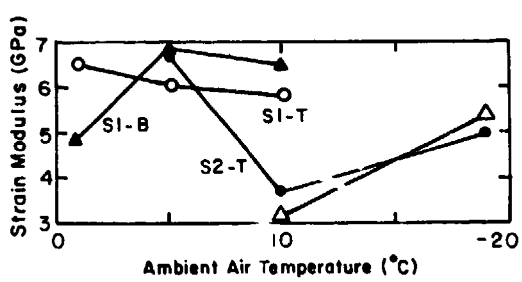


Figure 22. Strain modulus versus temperature of simply supported beams tested in conjunction with cantilever beams. Symbols T and B refer to top and bottom in tension tests respectively.

tively. Weighted averages (obtained by combining average values of individual batteries of tests) are plotted as a function of temperature in Figure 21 (flexural strength) and Figure 22 (strain modulus). Tests in which failure occurred at distances of greater than 10 cm from the transverse loading bar (greater than 10% of the beam span) were excluded from the averaged values listed in Tables 5 and 7. These represented less than 5% of the total number of beams tested.

Tests of columnar type (S2) ice showed no systematic variation of flexural strength with temperature or temperature gradient for either top or bottom in tension tests. This behavior essentially parallels that observed with cantilever beams. However, on average, S2-T (top in tension) beams tested 50 to 100% stronger than S2-B (bottom in tension) beams. Such a difference in strength is attributed primarily to grain size effects, mean cross-sectional diameters of crystals at the tops of S2 ice sheets being on the order of five times smaller than at the bottom. For example, even at the -1 °C test temperature where the beams are practically isothermal (-1 °C at the top, 0°C at the bottom), S2-T beams were on the order of 60% stronger than S2-B beams.

Tests of macrocrystalline S1 type ice also showed little if any systematic dependence on the temperature of the fiber in tension. This was particularly true of S1-T beams, which averaged 1200–1300 kPa over the entire range of test temperatures. However, these strengths are about 40% higher than those measured on S2-B beams despite the fact that the average cross-sectional diameter of crystals in the tension fiber of S2-B beams is an order of magnitude smaller than that of S1-T beams. Other factors being equal, small grain size should have led to greater flexural strength. The apparently contrary behavior observed in S1 ice is attributed to the failure characteristics of S1 ice per se, in which the majority of crystals exhibit vertical to near-vertical c-axis orientation. This, the crystal orientation effect, is even more pronounced in the case of S1-B tests, where the failure plane was often found to intersect as few as one or two crystals. Such behavior, in essence, approximates single crystal failure, in which the fracture plane is forced to propagate parallel to the direction of c-axis alignment, which also parallels the "hard fail" plane of single ice crystals. Not infrequently, failed S1 ice beams exhibited conchoidal fracture surfaces, rather than the vertical planar type fracture surfaces observed with S2 type ice beams.

Overall, parallel simply supported beams tested stronger than the corresponding beams tested in the cantilever mode. This was especially true of S2 ice tested with top in tension, in which the strength difference ratios of simply supported beams to cantilever beams averaged around 2.0, i.e., simply supported beams were approximately twice as strong as the same beams tested in the cantilever mode (Table 6). Other strength difference ratios listed in Table 6 varied between 1.03 and 1.44. These results are especially significant in regard to our earlier tests on cantilever beams, in which the normally sharp corners at the roots of the beams were modified by drilling 20-cm-diameter holes to provide relief from stress concentrations (see Table 3). A comparison of both sets of data (Tables 3 and 6) supports the contention that significant stress concentrations do exist at the roots of conventional cantilever beams. In macrocrystalline S1 type ice, the drilling of holes at the beam roots appears to substantially relieve stress risers at these locations. This situation applies to both top and bottom in tension tests and also to S2 type ice tested with the bottom in tension. However, in the case of S2 ice beams tested with the *top in tension*, it would appear that, despite drilling relief holes at the roots of the beams, stress riser effects still dominate tensile behavior in filleted beams. Otherwise, other factors, in addition to stress concentrations, need to be invoked to explain why simply supported beams tested with top in tension are 60–70% stronger than filleted cantilever beams with identical structural and thermal characteristics. Temperature gradients cannot be a factor since most of the tests were conducted at -1 °C ambient air temperature.

Table 6. Strength difference ratios of simply supported beams and cantilever beams tested in parallel.

Test temperature (°C)	*Seeded(S2)*		*Unseeded(S1)*	
	*Top**	*Bottom*	*Top**	*Bottom*
- 1	2.18	1.26	1.31	1.13
- 5	2.16	1.40	1.44	1.03
-10	1.85	1.20	1.37	1.21
-19	1.80	1.17	1.32	1.21

* Tension surface.

Lavrov (1971) would attribute such differences in the behavior of simply supported and cantilever beams to fundamental differences in their mechanics of failure. Lavrov further acknowledged that the bending strength of a simply supported beam should exceed that of the cantilever beam. He also determined, mainly from large beam tests, that the bending (flexural) strength of an ice cover can be obtained from cantilever beam tests by simply multiplying the latter by a correction factor of 1.5. This kind of strength difference factor is in good agreement with results reported here for S1 ice and for S2 ice if top and bottom in tension tests are averaged (Lavrov made no clear distinction between push-down and pull-up tests in reporting his results). Similar strength difference ratios have also been reported for large lake ice beams by Gow et al. (1978).

Our measurements of the flexural strength of freshwater model ice are also of interest with respect to urea ice, used for modeling sea ice, and for sea ice itself. According to Timco (1985) there is no apparent difference in flexural strength be-

Table 7. Average strain modulus (GPa) of parallel simply supported beams.

Ambient temperature (°C)	Seeded ice sheets *No. 1*	*No. 2*	*No. 4*	*No. 7*	*No. 8*	Unseeded ice sheets *No. 3*	*No. 5*	*No. 6*	Average *Seeded*	*Unseeded*
- 1						5.6*	7.7			6.5 ± 1.5
						5.1†	3.0			4.9 ± 1.0
- 5		6.7				4.7	8.4		6.7 ± 2.3	6.1 ± 2.0
		—				5.7	8.3			6.8 ± 1.7
-10		2.9	5.0			5.4	6.7		3.7 ± 1.3	5.8 ± 1.5
		2.7	4.5			5.2	8.0		3.2 ± 1.0	6.6 ± 1.6
-19			5.0						5.0 ± 1.6	
			5.4						5.4 ± 1.1	
Total beams		18	10			41	24		28	65

* Top in tension.
† Bottom in tension.

tween either cantilever or simply supported beams of urea ice or sea ice. This behavior is attributed to the widespread occurrence of brine (urea) inclusions and air pockets in the ice that effectively relieve stress concentrations through plastic flow. This explanation is compatible with that of Gow et al. (1978) regarding thermally modified lake ice in which the structure of the ice becomes sufficiently degraded, through the combined action of elevated air temperatures and solar radiation, to reduce intrinsic flexural strengths to levels less than the stress needed to activate stress risers at the roots of cantilever beams. However, since most of the data on sea ice were obtained from warm ice, we might suspect stress concentrations to develop in beams of cold sea ice containing fewer or smaller brine pockets.

Measurements of strain modulus on a total of 115 beams (Table 7) show no definitive trends. Values vary between 5 and 7 GPa except for those obtained at −10°C on a battery of seeded (S2) ice beams, which tested low (3.7 and 3.1 GPa) relative to other tests in this series, and with those obtained on the same beams tested in the cantilever mode.

Isothermal simply supported beams

A total of 252 individual beams was tested in this series of measurements. Detailed results of isothermal beam tests are included in Appendix A. Averaged values of flexural strength and strain modulus are listed in Tables 8 and 9 respectively. Weighted averages, plotted as a function of temperature, are presented in Figure 23 (flexural strength) and Figure 24 (strain modulus). As with the parallel simply supported beams, all tests with off-center breaks exceeding 10 cm were excluded from the averaged values listed in Tables 8 and 9. These represented less than 5% of the 252 beams tested.

Of the three types of tests performed in the current series of measurements, those involving isothermal simply supported beams yielded the clearest information concerning the effect of grain size, crystal orientation and temperature of the fiber in tension on the flexural characteristics of the ice. Isothermal beams tested the strongest overall. With regards to S2 type ice, all test series showed a substantial dependence of strength on the grain size of the fiber in tension (Fig. 23a). Significant increases in strength with decreasing temperature were also observed. Those beams tested with top in tension increased in average strength from about 1650 kPa at −1°C to nearly 2600 kPa at −19°C. In bottom in tension tests of S2 ice, flexural strength increased from about 1150 kPa at −1°C to 1640 kPa at −19°C. In short, the flexural strength of S2-B beams at −19°C is only about equal to the strength of S2-T beams at −1°C. Also, the ratios of strength for top and bottom in tension at the four test temperatures remain remarkably constant at 1.5, which agrees closely with the value obtained by Gow et al. (1978) on large beams of lake ice. Such differences in the flexural

Table 8. Average flexural strengths (kPa) of isothermal simply supported beams.

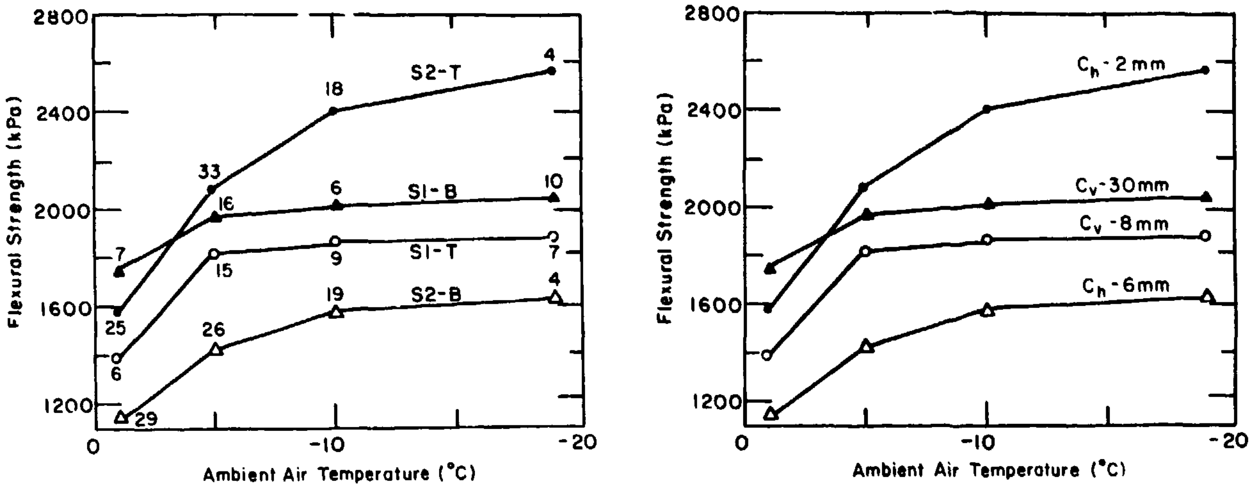
Ambient temperature	Seeded ice sheets						Unseeded ice sheets			Average	
(°C)	*No. 1*	*No. 2*	*No. 4*	*No. 6*	*No. 7*	*No. 8*	*No. 3*	*No. 5*	*No. 6*	*Seeded*	*Unseeded*
- 1			2092*			1573	1186	1479		1651 ± 369	1381 ± 194
			1268†			1054	1366	2237		1124 ± 131	1739 ± 487
			1979			1394					
			1154			1090					
- 5	1629	1630	2321	2208	2184		—	1849	1815	2101 ± 384	1824 ± 220
	1190	1324	1922	1388	1549		1394	1852	2028	1392 ± 265	1933 ± 313
			2214								
			1241								
-10		1495	2608		2329		1739	1925		2411 ± 289	1863 ± 150
		1273	1674		1598		1793	2102		1588 ± 166	1999 ± 326
-19			2572				1703	2022		2572 ± 285	1885 ± 207
			1641				2063	2026		1641 ± 122	2042 ± 184
Total beams	11	7	54	17	33	36	19	35	22	158	76

* Top in tension.
† Bottom in tension.

Table 9. Average strain modulus (GPa) of isothermal simply supported beams.

Ambient temperature	Seeded ice sheets						Unseeded ice sheets			Average	
(°C)	*No. 1*	*No. 2*	*No. 4*	*No. 6*	*No. 7*	*No. 8*	*No. 3*	*No. 5*	*No. 6*	*Seeded*	*Unseeded*
- 1			7.1*			6.8	5.2	8.5		6.9 ± 1.4	7.4 ± 2.1
			7.1†			7.0	5.0	6.9		7.0 ± 1.0	5.7 ± 1.1
- 5		4.6	5.6		6.2			7.9	7.0	5.9 ± 0.7	7.3 ± 0.8
		5.4	6.5		6.0			8.2	7.3	6.4 ± 1.0	7.6 ± 1.2
			5.8	6.0							
			6.6	6.9							
-10		4.4	5.4		6.8		5.6			6.4 ± 1.2	5.6 ± 0.4
		3.8	4.4		6.9		5.7			6.0 ± 1.5	5.7 ± 0.5
-19			8.3				7.0			8.3 ± 2.3	7.0 ± 1.2
			7.9				6.5			7.9 ± 0.9	6.5 ± 0.7
Total beams		6	36	16	28	31	19	15	19	117	53

* Top in tension.
† Bottom in tension.



a. Ice type (number of beams tested at each temperature is also indicated).

b. Crystal size/c-axis orientation relationships in S1 and S2 ice types.

Figure 23. Variation with temperature of the flexural strength of isothermal simply supported beams of S1 and S2 ice. Symbols T and B refer to top and bottom in tension tests respectively.

strength of isothermal beams of S2 ice are primarily the result of changes in grain size between the tops and the bottoms of the ice sheets. Typically, crystal cross-sectional diameters in S2 ice grown in the CRREL test tank increased from 1–2 mm near the top to 6–7 mm at the bottom.

In contrast to S2 ice, the differences in the strength between S1-T and S1-B beams are very much less and flexural strength became essentially independent of the thermal condition of the ice once temperatures had decreased below –5°C. The actual strength values of S1 ice fall more or less midway between those obtained with S2 ice, with values obtained at temperatures below –5°C averaging around 2000 kPa for S1-B beams and about 1800 kPa for S1-T beams. This represents a less than 10% difference in strength between S1-T and

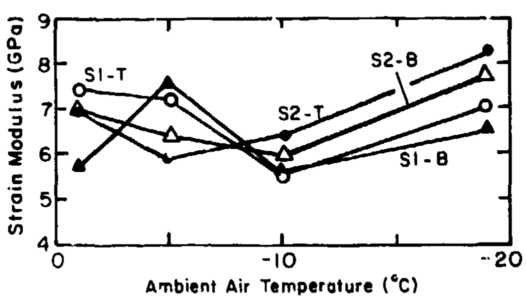


Figure 24. Strain modulus versus temperature of isothermal simply supported beams. Symbols T and B refer to top and bottom in tension tests respectively.

S1-B tests. Figure 23b, showing crystal size–crystal orientation relationships in S1 and S2 ice types, clearly demonstrates the importance of c-axis orientation relative to grain size effects. As noted earlier, grain size difference is the dominant factor controlling strength in S2 type ice. However, a crossover to the crystal orientation factor is evident when S2-B test results are compared with those obtained with S1-T and S1-B beams. Despite its finer-grained texture, the flexural strength of S2 ice tested with bottom in tension is appreciably weaker (on the order of 25–50%) than the much coarser-grained ice of either S1-T or S1-B beams.

In many S1-B tests, for example, failure occurred in an essentially single crystal mode, in a plane containing the c-axis and, hence, parallel to the "hard fail" plane of the ice crystal. The importance of the oriented crystal factor is further highlighted in the greater strength of beams of S1-B ice relative to S1-T beams, despite the fact that crystal size is appreciably smaller at the top of the ice sheet (in ice of S1-T beams). It is important to remember that temperature is not a factor here since the beams were isothermal.

Two beams of S2 ice were tested with sides in tension. Though too few to be statistically significant, the two beams tested much weaker than other isothermal beams; approximately 40% weaker than top in tension tests and 20% weaker than bottom in tension tests.

No attempt was made in the current series of tests to simulate fine-grained snow ice but, based on results obtained with fine-grained, seeded (S2) ice when it is loaded to tensile failure, we could expect cold, simulated snow ice to test as strong or stronger than the cold, seeded topmost layer of S2 ice sheets.

Strain modulus measurements were made on a total of 169 beams used in isothermal tests. Given the error limits of the deflection measurements, the strain moduli (Fig. 24) show no systematic pattern of variation with respect to changes in either the temperature of the ice or the magnitude of the flexural strength. Average values ranged from 6-8 GPa or about 10-20% higher than modulus data obtained with cantilever and simply supported beams tested in parallel with cantilever beams. Lavrov (1971) reports strain moduli of 67,000 kg/cm² (6.6 GPa) for S1 ice tested at -3 to -4°C and 39,000 kg/cm² (3.8 GPa) for S2 ice. According to Lavrov, strain modulus (E_f) should increase approximately linearly with increasing flexural strength (S_f) of the ice, the actual rate of increase depending on the rate of loading. Using data from tests on beams of S2 ice, Lavrov obtained the analytical relationship

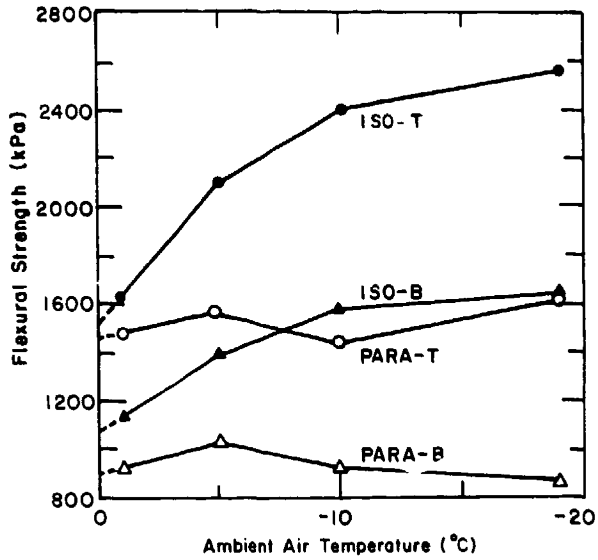
$$E_f = 3000\ S_f$$

for load durations of 3 to 4 seconds. We observed no such relationship in our tests in which beams were loaded to failure in less than 1 second.

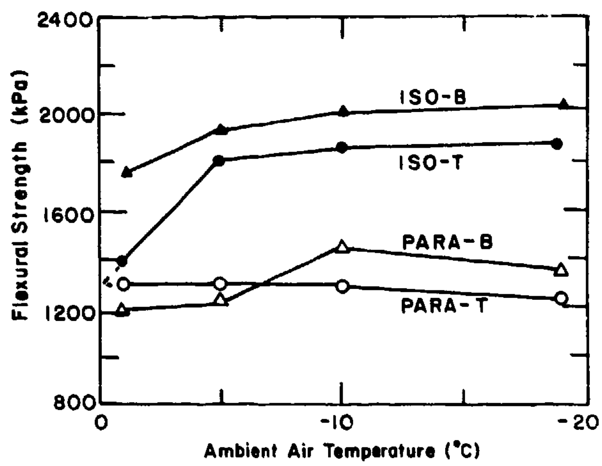
Temperature gradient effects

A second reason for conducting isothermal beam tests was to investigate the effects on flexural strength of temperature gradients in ice beams via comparisons with parallel simply supported beams of identical crystalline structure (either from the same ice sheet or from other ice sheets with the same structure). The two groups of beam tests differ only in their thermal state, the temperatures of isothermal beams remaining constant throughout their thickness whereas in parallel simply supported beams (those tested in conjunction with in-situ cantilever beams), only the top is at the prevailing ambient temperature, with the bottom of the beam being at or very close to 0°C. Accordingly, the only differences in the flexural characteristics of the two kinds of simply supported beams should be directly linked to the effect of temperature gradients in the parallel simply supported beams.

This effect can be a very substantial one, especially in a 10-cm-thick ice beam at an ambient air temperature of -19°C. As an inspection of Figure



a. S2 ice.



b. S1 ice.

Figure 25. Comparative relationships of flexural strengths of isothermal and parallel simply supported beams, demonstrating temperature gradient effects. Extrapolations of data to 0°C are indicated by dashed lines.

25 shows, the net effect of temperature gradients is to significantly reduce the strength of parallel simply supported beams, relative to isothermal beams tested at corresponding ambient air temperatures. This situation applies to both S1 and S2 ice types. In S2 ice, for example, the flexural strengths of parallel simply supported beams at the lower ambient air temperatures (larger temperature gradients) decreased to less than 55% of those of the isothermal beams. The effect is somewhat less in S1 ice beams, which tested about 30-35% weaker than their isothermal counter-

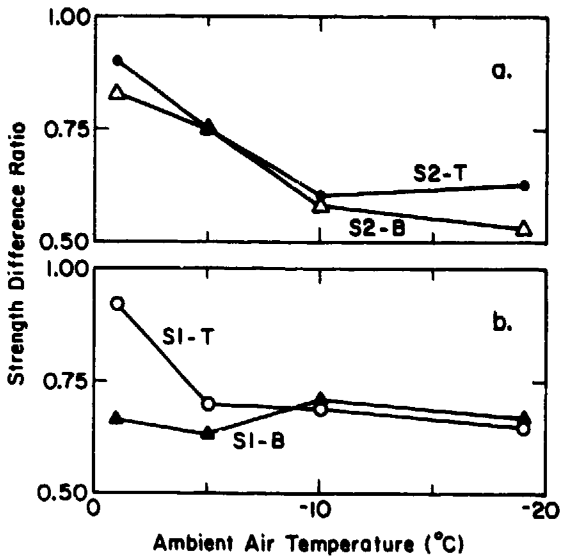


Figure 26. Strength difference ratios (flexural strengths of parallel simply supported beams relative to those of isothermal beams) versus ambient air temperature for (a) S2 ice and (b) S1 ice. Symbols T and B refer to top and bottom in tension tests respectively.

parts. Relationships between the two beam groups, plotted as a strength difference ratio versus ambient air temperature and ice type, are shown in Figure 26. It is also interesting to note in both Figures 25 and 26 that as ambient surface air temperatures converge towards 0°C so do the strengths of the corresponding sets of beams, e.g., ISO-T and PARA-T. This is to be expected since at 0°C both isothermal and parallel simply supported beams are now thermally equivalent. The only exception appears to be with ISO-B and PARA-B tests of S1 ice.

Notwithstanding, the fact that this convergence occurs in three out of four sets of tests lends credence to the testing procedures employed in the current series of tests and gives us confidence in the conclusion that the *existence* of a temperature gradient, regardless of the temperatue of the fiber in tension, is ultimately determining the strengths of parallel simply supported beams. However, as noted earlier in the discussion of the parallel beam test data, the ultimate strength of any particular set of parallel simply supported beams appears little affected by the *magnitude* of the temperature gradient. This seems a critical result since parallel beam tests come closest to approximating in-situ ice tests and are the ones most likely to yield realistic values of the effective flexural strength and modulus of floating freshwater ice sheets. In the cases of tests of structurally undegraded freshwater model ice sheets in the CRREL tank, S2 ice with top in tension would appear the strongest (1500 kPa) and S2 ice with bottom in tension (900 kPa) the weakest, with S1 ice values (1300 kPa) falling in between. This result would need to be modified for natural ice covers close to their melting points, because of the associated solar-modulated degrading and candling of the ice crystal structure. Such degrading of natural ice sheets (not a factor in indoor test tanks) is known to be a major cause of loss of flexural strength of temperate lake ice in spring (Weeks and Assur 1969, Gow et al. 1978).

Comparisons with other laboratory and field data

The results from small beam testing of freshwater model ice, in which the flexural strengths of simply supported isothermal beams were measured, include those of Lavrov (1971) and Timco and Frederking (1982). Timco and Frederking limited their measurements to S2 ice, which they tested at −10°C ambient air temperatures. Beams were tested in both push-down and pull-up modes. With push-down (top in tension tests) they obtained an average flexural strength of 2200 kPa, in good agreement with our averaged value of 2443 kPa at −10°C. Their value of 1770 kPa for bottom in tension (pull-up tests) also agrees reasonably well with our measurement of 1588 kPa. Lavrov (1971) tested both S1 and S2 ice and his results are reported in several tables and diagrams. Although the precise nature of load application is not given (Lavrov did not always distinguish between top and bottom in tension tests), indications are that results in his Table 38 (p. 114) were obtained with push-down (top in tension) tests. He lists mean values of 20.5 kg/cm² (2010 kPa) for S1 ice and 22.0 kg/cm² (2160 kPa) for S2 ice tested isothermally at -3 to -4°C. These data conform closely with our mean values of 1820 kPa for S1 ice and 2100 kPa for S2 ice tested with top in tension at -5°C.

Lavrov also furnished data on the temperature dependence (0 to −40°C) of the bending strength of S2 ice sheets grown in the laboratory. Though precise information on the nature of loading of the simply supported beams is lacking, we have assumed that both top and bottom in tension tests were performed. The part of his Figure 62 (p. 120) relating to freshwater ice tests is reproduced here (Fig. 27) to indicate the excellent agreement with

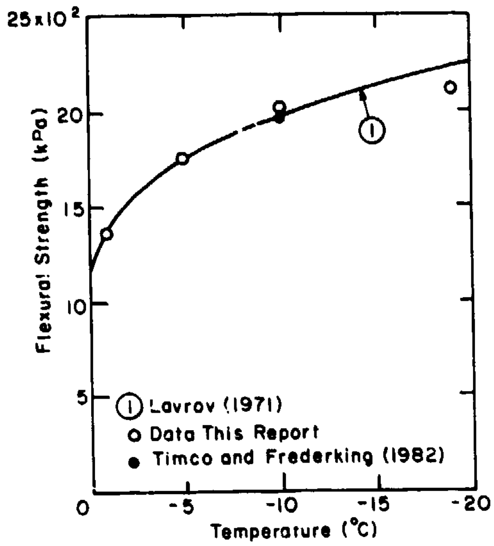


Figure 27. A comparison of data from several sources relating the flexural strength of isothermal simply supported beams to the temperature of the ice. All data based on small beam testing of S2 type ice sheets grown in laboratory test tanks.

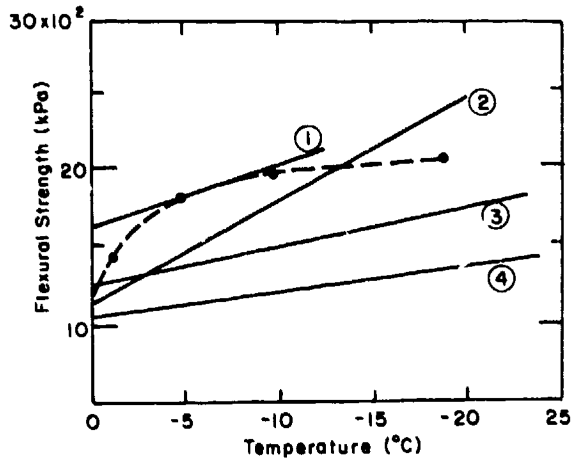


Figure 28. Flexural strength data of isothermal beams from the current series of tests (dashed curve) compared with small beam tests on lake and river ice. Diagram is adapted from Weeks and Assur (1969). Data sources are: 1) Frankenstein (1959), 2) Voitkovskii (1960), 3) Wilson and Horeth (1948), Brown (1926) and Hitch (1959), 4) Butyagin (1966).

data (averaged values of top and bottom in tension tests) obtained in the current series of tests.

The above comparisons were all made with reference to freshwater model ice tests conducted under conditions very similar to those in the CRREL tank. However, for completeness, a comparison with results from tests on simply supported ice beams from a number of diverse field locations would seem in order. In their review of the literature on the fracture of lake and sea ice, Weeks and Assur (1969) devoted some space to discussing flexural strength measurements of small beams. Their Figure 39 (p. 49) summarizes data from a variety of sources plotted in terms of the flexural strength versus temperature. A modified version of their Figure 39 is reproduced here as Figure 28. Although all four data sets vary widely, for reasons unknown, they do nevertheless indicate a trend towards increasing flexural strength with decreasing temperature, similar to results reported here for simply supported isothermal beams. Our data set (averaged on the basis of all tests on S1 and S2 ice) is also plotted in Figure 28. It perhaps conforms closest with results obtained by Frankenstein (1959) and Voitkovskii (1960). Reasons for the disparate nature of much of the flexural strength data are not clear, though Weeks and Assur (1969) suggest it may be related to structural differences in the ice that was tested, details of which were either omitted or not sufficiently documented in the original reports. Structural differences could be either due to differences in original growth textures (S1 or S2 type ice) or related to thermal modification of the ice. This situation, in conjunction with results reported here on model freshwater ice, simply points up the need for researchers to diligently document the structural and thermal characteristics of the ice they are testing if realistic evaluations of the mechanical properties of ice sheets are to be obtained.

CONCLUSIONS

Previous investigations (Gow et al. 1978) of the flexural strength of large beams of lake ice showed that the strength of the ice depends appreciably on its temperature and crystalline composition. This work has now been extended to studies under controlled conditions in a refrigerated test tank where measurements were made on freshwater model ice sheets corresponding in structure to the two major ice types, S1 and S2, encountered in frozen lake water. S1 ice, produced by spontaneous nucleation of water at the freezing point, yields macrocrystalline ice sheets with predominantly vertical c-axes. S2 ice is produced when the surface of the water is seeded prior to freezing; structurally it consists of vertically elongated, columnar crystals with predominantly horizontal c-axes. Using a combination of beam tests, utilizing cantilever as well as simply supported beams, we have been successful in determining independently the effects of grain size, crystal orientation, tension fiber temperature and temperature gradient on the flexural characteristics of freshwater model ice. Results of major interest can be summarized as follows:

1. Comparative tests of in-situ cantilever beams and of the same beams tested immediately afterwards in three-point loading (so as to preserve in-situ temperature profiles) showed that the sharply terminated roots of conventional cantilever beams activated a significant stress concentration and caused premature failure. This situation can be relieved and (in most cases) substantially eliminated by drilling 10-cm-radius holes at the roots of the beams. A further significant feature of these combination beam tests was the virtual non-dependence of flexural strength of the beams on the temperature of the fiber in tension. This situation applied to both top and bottom in tension tests over the temperature range -1 to -19°C; strengths did not exceed 1200 kPa for conventional cantilever beams or 1650 kPa for the simply supported beams.

2. The highest flexural strengths were measured on isothermal beams tested in the simply supported mode. Of these, fine-grained columnar ice at the top of S2 ice sheets generally tested the strongest at any given temperature, with an average flexural strength of 1650 kPa at -1°C, increasing to nearly 2600 kPa at -19°C. However, five- to six-fold increases in the cross-sectional diameter of crystals at the bottom of S2 ice sheets resulted in very substantial decreases in flexural strength of isothermal beams tested with bottom in tension. Simultaneously, c-axis orientation effects began to exercise their influence, such that beams of S1 ice composed of very much larger crystals with predominantly vertical c-axes became appreciably stronger in flexure than the coarsest-grained S2 ice having crystals with predominantly horizontal c-axes.

3. Tests on isothermal beams also afforded direct verification of the loss of flexural strength associated with temperature gradients in parallel simply supported beams (those tested in parallel with cantilever beams). This is an important result because parallel simply supported beam tests, unlike those of conventional cantilever beams, are not affected by stress concentrations. Accordingly, they are the ones most likely to approximate true in-situ ice testing and should yield reasonably realistic values of the effective flexural strength of floating freshwater ice covers. However, cantilever beam tests are simpler to perform and, if suitably corrected for stress concentration effects on the basis of data from the parallel simply supported beam tests, should also yield reliable in-situ measurements of flexural strength. Correction factors of +30 to 35% were obtained for S1 ice with top and bottom in tension and for S2 ice with bottom in tension. For S2 ice with top in tension, a correction factor of approximately +100% is indicated by our results.

4. Experiments with beam dimension changes, including increasing the width by 60% with respect to the thickness, and with varying the length to thickness ratio from 7:1 to 10:1 resulted in no significant changes in flexural strengths of beams.

5. Strain moduli, calculated from beam deflection measurements, showed little dependence on either the temperature of the ice or the magnitude of the flexural strength. Average values ranged from 5 to 7 GPa for cantilever and parallel simply supported beams and from 6 to 8 GPa for isothermal simply supported beams.

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APPENDIX A: FLEXURAL STRENGTH AND STRAIN MODULUS MEASUREMENTS OF CANTILEVER AND SIMPLY SUPPORTED BEAMS OF FRESHWATER MODEL ICE

Symbols and notations: T and B beside beam numbers denote top and bottom in tension tests, respectively; L, w and h denote the length, width and thickness of beams, respectively; P is the failure load, d is the beam deflection, S is the calculated flexural strength and E is the calculated strain modulus.

SHEET 1
(SEEDED)

Feb. 28, 1983 (-5°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1	T	103.8	10.5	11.8	18.0	.102	760	4.5
2	T	104.1	10.5	11.8	19.4	.114	826	4.4
3	T	105.4	10.5	11.8	15.0	.089	643	4.5
4	T	105.4	20.3	11.4	27.8		664	
5	T	108.6	20.0	11.1	27.0	.117	696	4.2
6	T	105.4	19.4	10.8	25.5	.127	700	3.8
7	T	108.3	20.6	10.8	27.8	.140	734	3.8
8	T	108.6	19.7	10.8	24.0	.127	690	3.8

Ave. S_T = 714 ± 58
Ave. E_T = 4.1 ± 0.3

Mar. 1, 1983 (-5°C) Cantilever

9								
10	T	102.2	12.1	9.8	12.7	.097	657	4.8
11	T	101.6	12.1	10.0	15.0	.109	742	4.7
12	T	101.3	12.1	9.8	15.0	.112	746	4.8
13	T	101.6	12.1	9.8	14.6	.122	750	4.3

Ave. S_T = 724 ± 45
Ave. E_T = 4.7 ± 0.2

Mar. 1, 1983 (-5°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
14	B	101.6	11.4	10.5	90.0		1064	
15	T	101.6	12.1	10.5	131.9		1554	
16	T	101.6	11.4	10.5	123.8		1498	
17	B	101.6	12.1	10.5	97.7		1226	
18	B	101.6	11.9	10.4	91.4		1116	
19	T	101.6	11.8	10.2	126.0		1552	
20	B	101.6	11.6	10.5	97.7		1216	
21	S	101.6	10.5	11.4	100.4		937*	
22	S	101.6	10.3	11.4	90.0		953*	
23	B	101.6	10.2	11.0	81.0		1041	
24	T	101.6	10.2	11.1	136.3		1561	
25	B	71.1	10.5	6.4	61.7		1475	
26	T	71.1	10.5	6.4	77.9		1979	

Ave. S_T = 1629 ±197; Ave. S_B = 1190 ±159

*Sideways Test

SHEET 1 (Cont.)
(SEEDED)

Mar. 4, 1983 (-3°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
27	T	111.8	11.3	10.3	10.9	.107	596	4.5
28	T	111.8	11.1	10.3	11.0	.112	612	4.4
29	B	111.8	11.6	10.3	13.6	.147	726	4.0
30	B	109.2	11.8	10.3	11.3	.114	584	3.9
31	B	110.5	11.4	10.1	12.1	.127	688	4.4
32	T	111.8	11.4	10.0	11.2	.125	643	4.3

Ave. S_T = 617 ± 24; Ave. S_B = 666 ± 24
Ave. E_T = 4.4 ±0.1; Ave. E_B = 4.1 ±0.3

Mar. 7, 1983 (-17°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
33	B	101.6	10.0	10.4	94.5		1294	
34	T	101.6	11.1	10.1	144.0	.041	1883	7.9
35	T	101.7	11.3	10.2	147.2	.053	1542+	5.9
36	B	101.6	10.8	10.0	84.2	.028	1089	7.2
37	T	101.6	11.4	10.1	147.2	.051	1901	6.3
38	T	101.6	11.1	10.2	147.2	.038	1687	8.4
39	B	101.6	11.5	9.9	95.9	.028	1342	7.9

Ave. S_T = 1824 ±119; Ave. S_B = 1242 ±134
Ave. E_T = 7.6 ±1.1; Ave. E_B = 7.6 ±0.5

+Off center break exceeding 10 cm.

SHEET 2
(SEEDED)

Mar. 28, 1983 (-5°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1		RECORDER MALFUNCTION						
2	T	97.0	10.4	11.3	19.3	.117	828	3.9
3	T	97.0	10.4	11.2	14.2	.089	626	3.9
4	T	96.7	10.0	11.0	11.3	.074	529	4.1
5	T	BROKE AT CRACK						
6	T	95.7	11.1	10.9	14.4	.099	613	3.5
7	T	95.3	10.3	10.8	12.5	.097	579	3.4
8	T	95.3	9.3	10.6	14.4	.125	771	3.5
9	T	96.5	10.2	10.9	14.0	.112	655	3.3
10	B	BROKEN						
11	B	BROKEN						
12	B	94.5	10.9	11.0	14.3	.066	616	4.9
13		BROKEN						
14	B	95.5	10.5	11.0	16.0	.079	696	5.0
15	B	95.0	10.5	11.0	16.8	.086	727	4.7
16	B	95.5	9.2	10.9	15.4	.089	779	5.0
17	T	96.0	9.5	10.8	14.6	.084	731	5.0
18	T	96.0	11.5	10.8	17.6	.089	730	4.7

Ave. S_T = 673 ± 97; Ave. S_B = 705 ± 68
Ave. E_T = 3.9 ±0.6; Ave. E_B = 4.9 ±0.1

Mar. 29, 1983 (-5°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1	T	101.6	10.4	11.1	135.0	.053	1629	4.6
2	T	71.1	10.0	11.0	184.5		1630	
3	T	71.1	10.0	11.0	132.8		854+	
11	B	71.1	9.8	10.6	135.0	.023	1316	4.4
12	B	71.1	10.0	10.1	152.1		1278	
13	B	71.1	9.8	10.6	150.3	.018	1380	6.3

Ave. S_T = 1630 ±1; Ave. S_B = 1324 ± 52
Ave. E_T = 4.6 ; Ave. E_B = 5.4 ±1.3

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1		Used for Isothermal Tests, Mar. 29						
2		"			"			
3		"			"			
4	T	71.1	10.1	10.8	135.0		1229	
5		BROKEN						
6	T	71.1	11.1	10.9	156.6		1272	
7	T	71.1	9.9	10.6	165.2		1584	
8	T	71.1	10.0	10.5	108.9		712 +	

Ave. S_T = 1361 ±194

Beams 11, 12 and 13 used for Isothermal Tests, Mar. 29
Other beams not tested.

+Off center break exceeding 10cm

SHEET 2 (Cont.)
(SEEDED)

Mar. 29, 1983 (-5°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
19		BROKEN						
20	B	105.5	11.5	11.8	17.9	.173	711	2.5
21	B	107.0	11.2	11.7	17.6	.086	723	5.5
22	B	107.0	11.4	11.9	18.2	.081	708	5.6
23	B	107.5	11.0	12.0	19.0	.097	757	5.0
24	B	106.5	11.0	12.0	17.4	.097	685	4.5
25	T	109.0	11.2	12.0	17.6	.071	700	6.5
26	T	109.5	11.6	12.0	20.9		805	
27	T	108.5	12.2	12.0	20.1	.097	728	4.9
28	T	109.0	11.8	12.0	22.8	.127	859	4.5
29	T	109.0	11.5	11.8	19.5	.104	788	5.0
30	T	109.0	11.7	11.5	19.0	.109	786	5.0

Ave. S_T = 778 ± 57; Ave. S_B = 717 ± 26
Ave. E_T = 5.2 ±0.8; Ave. E_B = 4.6 ±1.3

†Faulty experimental technique, exclude.

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
19	B	101.6	11.5	12.1	104.4	.015	988	8.8 †
20	B	101.6	11.3	12.2	117.5		1092	
21	B	101.6	11.0	12.0	113.0	.013	1070	11.8 †
22	B	101.6	11.5	12.6	117.5		1024	
23		BROKEN						
24		BROKEN						
25	T	101.6	11.6	12.3	217.4	.028	1906	9.2
26	T	101.6	11.8	12.3	185.0	.038	1604	5.7
27	T	101.6	11.9	12.1	217.4	.053	1921	5.0
28		BROKEN						
29		BROKEN						
30		BROKEN						

Ave. S_T = 1810 ±179; Ave. S_B = 1043 ±47
Ave. E_T = 6.7 ±2.3

Mar. 31, 1983 (-1°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
31	T	110.0	11.5	12.0	19.5		761	
32	T	109.0	11.5	12.0	14.9		578	
33	T	110.0	12.0	11.9	16.1		619	
34	T	110.0	11.8	11.8	16.3		640	
35	T	110.0	11.5	11.4	13.6		589	
36	T	109.5	11.5	11.5	15.9		672	
37	T	109.5	12.1	11.7	15.1		588	
38	T	109.5	11.4	11.7	12.7		528	
39	B	108.5	11.8	11.7	17.8		702	
40	B	108.5	11.9	11.5	18.9		767	
41	B	109.0	12.1	11.2	19.3		814	
42	B	111.0	10.7	11.0	12.7		639	
43	B	111.0	12.8	11.0	14.2		597	
44	B	.11.0	11.5	11.0	12.7		594	
45	B	111.0	10.5	10.2	13.2		709	

Ave. S_T = 622 ±71; Ave. S_B = 689 ±84

+Off center break exceeding 10 cm.

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
31		Used in Isothermal Beam Tests, Apr. 1						
32	T	101.6	11.7	12.3	139.5		850 +	
33	T	101.6	11.8	11.9	147.2		1373	
34	T	101.6	11.7	11.8	149.9		1462	
35	T	101.6	11.6	11.9	157.5		1490	
36	T	101.6	12.1	11.9	165.2		1497	
37	T	101.6	11.8	11.8	161.1		1525	
38		Used in Isothermal Beam Tests, Apr. 1						
39		" " " "						
40	B	101.6	12.2	11.4	71.1		733	
41	B	101.5	11.5	11.4	84.2		916	
42	B	101.6	11.2	11.1	85.5		907	
43	B	101.6	11.9	11.1	70.7		827	
44		Used in Isothermal Beam Tests, Apr. 1						
45		" " " "						

Ave. S_T = 1469 ±58; Ave. S_B = 846 ±85

Apr. 1, 1983 (-10°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
31	T	101.6	11.4	12.4	217.4		1180 +	
38	T	101.6	11.8	11.5	148.5		1049 +	
39	B	101.6	11.5	11.5	131.9	.058	1357	3.3
44	B	101.6	11.5	10.7	106.7	.046	1190	4.2
45	T	101.6	11.3	10.6	121.5	.053	1495	4.4

Ave. S_T = 1495; Ave. S_B = 1273 ±118
Ave. E_T = 4.4; Ave. E_B = 3.8 ±0.6

Parallel Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)

Apr. 1, 1983 (-10°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
46	T	110.0	11.8	11.2	16.7		727	
47	T	110.0	11.5	11.0	16.3		756	
48	T	110.0	11.5	11.0	17.4	.114	808	5.2
49	T	110.0	11.9	11.0	16.7	.109	747	5.1
50	T	110.5	11.3	11.3	14.6		652	
51		BROKEN						
52	T	110.5	11.9	11.8	17.4		682	
53	T	110.5	11.6	11.5	19.9	.102	772	5.9
54	B	109.5	11.7	12.2	15.5		573	
55	B	108.5	11.6	12.2	18.9		699	
56		BROKEN						
57	B	108.5	11.3	12.2	20.8	.099	789	5.1
58	B	109.5	10.8	12.0	19.9		752	
59	B	108.5	11.9	12.1	20.8		762	
60	B	108.5	11.7	12.1	20.3	.099	757	5.0
61	B	108.0	11.4	11.9	21.3		837	
62	B	108.0	11.6	11.9	19.4	.099	749	5.0

Ave. S_T = 734 ± 53; Ave. S_B = 739 ± 78
Ave. E_T = 5.4 ±0.4; Ave. E_B = 5.0 ±0.1
+Off center break exceeding 10 cm.

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
46	T	101.6	11.8	11.7	300.0		2833 +	
47	T	101.6	11.7	11.8	210.2		1985	
48	T	101.6	12.0	11.5	161.1	.112	1578	2.0
49	T	101.6	11.4	11.5	142.7	.097	1472	2.2
50	T	101.6	12.1	11.8	183.6	.053	1688	4.5
51	T	101.6	11.5	11.9	180.0	.102	1417	2.3
52	T	101.6	12.0	12.2	225.0	.066	1391 +	4.0
53	T	101.6	12.0	12.3	131.4	.041	988	3.7
54	B	101.6	11.9	12.0	108.9	.064	950	2.1
55	B	101.6	11.8	12.2	113.9	.046	970	3.0
56	B	101.6	12.0	12.3	85.5	.038	728	2.6
57	B	101.6	12.0	12.6	117.0	.041	973	3.1
58	B	101.6	12.1	12.5	111.6	.046	980	2.6
59	B	101.6	11.9	12.3	103.5	.036	890	3.3
60	B	101.6	11.8	12.0	121.5	.061	1126	2.5
61	B	101.6	11.8	12.2	85.5	.036	663	2.9
62	B	101.6	11.8	12.3	90.0	.048	809	2.2

Ave. S_T = 1521 ±330; Ave. S_B = 899 ±143
Ave. E_T = 2.9 ±1.1; Ave. E_B = 2.7 ±0.4

SHEET 2 (Cont.)
(SEEDED)

Apr. 4, 1983 (-19°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(	d(cm)	S(kPa)	E(GPa)
63	T	110.0	11.4	12.8	19.9		688	
64	T	110.5	11.7	13.0	23.6		777	
65	T	BROKEN						
66	T	108.5	11.1	13.0	10.4		352 +	
67		RECORDER MALFUNCTION						
68	T	111.0	12.4	13.0	30.7		957	
69	T	109.5	11.2	12.9	26.6		865	
70	T	110.0	12.2	12.7	30.2		1010	
71	B	109.2	12.0	13.3	19.4		586	
72	B	110.0	12.0	13.1	23.6	.091	742	5.0
73	B	110.2	11.8	13.3	24.1		748	
74	B	110.0	11.9	13.1	21.3	.084	674	5.0
75	B	109.8	12.0	13.0	16.1		512	
76	B	110.0	11.6	12.8	22.7		772	
77	B	110.5	11.8	12.7	21.3		726	
78	B	110.0	11.8	12.5	19.4		680	

Ave. S_T = 859 ±131; Ave. S_B = 680 ±90
Ave. E_B = 5.0 ± 0
+Off center break exceeding 10 cm.

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
63		BEAMS BROKEN PRIOR TO TESTING						
64		"	"	"	"			
65		"	"	"	"			
66		"	"	"	"			
67	T	101.6	11.9	13.2	161.1		1213	
68	T	101.6	11.9	13.2	255.2		1889	
69	T	101.6	11.6	13.3	172.4		1308	
70	T	101.6	12.0	13.0	225.0		1738	
71	B	101.6	11.7	13.4	112.5		852	
72	B	BROKEN						
73	B	101.6	12.1	13.2	108.0		818	
74	B	101.6	11.8	13.0	90.0		728	
75	B	101.6	11.9	12.6	93.2		726	
76	B	101.6	11.8	12.1	100.4		884	
77	B	101.6	11.8	12.4	91.4		810	
78	B	101.6	11.6	12.4	104.9		935	

Ave. S_T = 1537 ±327; Ave. S_B = 822 ±77

SHEET 3
(UNSEEDED)

Apr. 11, 1983 (-5°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1	T	108.4	10.2	9.5	15.9	.127	1100	7.2
2	T	109.0	10.5	9.5	12.5	.114	843	6.2
3	T	110.0	9.7	9.2	11.2	.102	879	7.6
4	T	110.0	10.0	9.6	14.6	.127	1022	6.8
5	T	109.5	10.5	9.8	11.5	.102	736	5.9
6	T	109.0	10.0	9.7	12.9	.108	877	6.6
7	T	109.0	11.1	9.7	13.8	.111	848	6.2
8	T	108.0	10.1	9.7	11.9	.102	803	6.4
9	B	108.5	10.5	9.6	20.8	.165	1365	6.7
10	B	108.0	10.3	9.8	15.1	.124	975	6.3
11	B	108.0	10.0	10.0	18.0	.102	1140	8.7
12	B	108.0	10.9	10.0	24.2	.184	1410	6.0
13	B	108.0	9.6	10.0	16.5	.130	1094	6.5
14	B	108.0	10.2	10.2	17.0	.133	1018	5.8
15	B	108.0	9.7	9.9	19.4	.140	1295	7.3
16	B	108.5	10.3	10.0	16.6	.111	1026	7.3

Ave. S_T = 888 ±118; Ave. S_B = 1165 ±169
Ave. E_T = 6.6 ±0.6; Ave. E_B = 6.8 ±0.9

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1	T	101.6	10.2	9.5	79.5	.043	1324	5.4
2	T	101.6	9.9	9.7	78.0	.040	1253	5.5
3	T	101.6	9.7	9.9	75.0	.051	1249	4.0
4	T	101.6	10.0	10.0	75.0	.043	1068	4.5
5	T	101.6	10.5	9.9	85.5	.040	1262	5.4
6	T	101.6	10.8	10.0	75.0	.044	1107	4.1
7	T	101.6	11.0	10.0	84.0	.049	1188	4.0
8	T	101.6	10.1	10.1	90.0	.048	1270	4.6
9		RECORDER MALFUNCTION						
10	B	101.6	10.0	10.6	70.5	.019	919	8.0
11	B	101.6	10.7	10.5	88.5	.032	1187	5.7
12	B	101.6	10.4	10.5	82.5	.033	993	5.3
13	B	101.6	10.7	10.7	106.5	.035	1274	6.0
14	B	101.6	10.0	10.8	81.0	.035	1071	4.7
15	B	101.6	10.5	11.0	96.0		1192	
16	B	101.0	10.4	11.1	96.0	.038	1160	4.5

Ave. S_T = 1215 ± 88; Ave. S_B = 1114 ±125
Ave. E_T = 4.7 ±0.7; Ave. E_B = 5.7 ±1.3

Apr. 12, 1983 (-1°C) Isothermal Simple Support

17	B	101.6	10.8	9.5	75.0	.048	1223	4.3
18	T	101.6	11.2	9.3	70.5	.033	1162	6.1
19	B	101.6	11.4	9.3	88.5	.048	1389	5.2
20	B	101.6	10.0	9.3	67.5	.041	1229	5.3
21	B	101.6	10.5	9.4	99.0	.054	1622	5.4
22	T	81.3	10.6	9.6	97.5	.033	1209	4.2

Ave. S_T = 1186 ± 33; Ave. S_B = 1366 ±187
Ave. E_T = 5.2 ±1.3; Ave. E_B = 5.1 ±0.5

SHEET 3 (Cont.)
(UNSEEDED)

Apr. 12, 1983 (-1°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
23 T	107.0	10.6	10.5	18.9	.108	1023	6.9
24 T	102.0	10.5	10.5	18.9	.102	980	6.3
25 T	105.5	10.9	10.6	18.9	.098	958	4.5
26 T	106.0	11.1	10.7	16.1		788	
27 T	105.5	10.5	10.6	16.1	.098	843	4.0
28 T	108.0	10.5	10.3	16.7	.102	949	4.9
29 T	102.5	10.3	10.4	16.3	.105	880	5.7
30 T	105.5	11.6	10.3	24.6	.158	1239	5.7
31 B	104.5	10.0	10.0	17.8	.124	1092	6.4
32 B	102.0	9.6	9.9	17.5	.146	1185	5.4
33 B	104.5	10.7	9.6	16.6	.137	1031	5.7
34 B	105.0	9.5	9.9	15.6	.152	1034	5.1
35	BROKEN						
36 B	106.0	9.9	9.2	12.3	.130	914	5.7
37 B	104.5	10.2	9.1	15.3	.156	1114	5.7
38 B	88.0	10.5	9.1	15.3		911	

Ave. S_T = 958 ±137; Ave. S_B = 1040 ±102
Ave. E_T = 5.4 ±1.0; Ave. E_B = 5.7 ±0.4

Parallel Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
23 T	101.6	10.1	11.0	81.0	.024	1053	6.5
24 T	101.6	10.8	10.9	93.0	.032	1147	5.3
25 T	101.6	10.8	10.9	90.0	.035	988	4.7
26 T	101.6	11.3	10.9	96.0	.030	1026	5.6
27 B	101.6	10.9	11.0	114.0	.041	1227	4.9
28 T	101.6	10.3	10.8	77.9	.033	1034	4.1
29 T	101.6	10.7	10.6	90.0	.030	1052	6.1
30 T	101.6	11.7	10.5	100.5	.029	1177	6.6
31 B	101.6	10.0	10.5	72.0	.037	1042	4.3
32 B	101.6	10.3	10.0	78.0	.032	1045	6.1
33 B	101.6	10.5	9.9	84.0	.040	1290	5.3
34 B	101.6	10.2	9.6	84.0	.043	1407*	5.6
35	BROKEN						
36 B	101.6	10.1	9.5	54.0	.027	871	5.9
37 B	101.6	10.1	9.0	70.5	.046	1098+	5.3
38 B	81.3	10.5	9.2	78.0	.032	922	3.9

Ave. S_T = 1068 ± 68; Ave. S_B = 1066 ±165
Ave. E_T = 5.6 ±0.9; Ave. E_B = 5.1 ±0.8

Apr. 13, 1983 (-10°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
39 B	101.6	11.2	10.2	123.0	.038	1151+	7.0
40 T	101.6	10.9	9.8	130.5	.056	1786	5.9
41 T	101.6	11.1	9.4	108.0	.057	1737	5.2
42 B	101.6	11.3	9.4	123.0	.064	1914	5.3
43 B	101.6	11.4	10.2	126.0	.045	1671	6.0
44 T	81.3	12.0	10.2	172.5	.032	1695	5.6

Ave. S_T = 1739 ± 46; Ave. S_B = 1793 ±172
Ave. E_T = 5.6 ±0.4; Ave. E_B = 5.7 ±0.5

*Sideways Test
+Off center break exceeding 10 cm.

Apr. 13, 1983 (-10°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
45	BROKEN						
46 T	108.0	11.2	9.9	16.5	.130	952	[illegible]
47 T	105.0	11.6	9.6	13.6	.102	786	5.9
48 T	106.0	11.0	9.7	13.1	.095	794	6.4
49 T	106.5	11.6	9.6	12.9	.095	753	6.2
50 B	106.0	11.0	9.6	18.7	.140	1151	6.4
51 B	105.0	11.0	9.9	19.9	.156	1137	5.4
52 B	105.5	10.7	9.5	20.3	.165	1306	6.2
53 B	103.5	11.9	9.5	22.2	.175	1264	5.4
54 T	105.0	10.6	9.3	7.6	.061	509 +	6.7
55 T	106.5	10.6	9.1	11.7	.108	836	6.4
56 T	104.3	11.0	8.9	9.1		662	
57 T	106.5	11.6	8.8	13.1	.080	910	9.9
58 T	106.0	11.0	8.8	12.3		899	
59 B	106.0	10.9	8.9	16.1	.159	1160	6.2
60 B	106.5	10.2	8.9	14.4	.137	1114	6.9

Ave. S_T = 824 ± 95; Ave. S_B = 1188 ± 77
Ave. E_T = 6.8 ±1.6; Ave. E_B = 6.1 ±0.6

Parallel Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
45							
46 T	101.6	11.3	9.3	88.5	.029	1348	8.7
47 T	101.6	11.1	9.8	76.5	.033	1080	5.7
48 T	101.6	11.3	10.0	79.5	.052	1059	3.5
49 T	101.6	11.1	9.9	78.0	.033	1141	5.6
50 B	101.6	11.7	9.9	90.0	.041	1196	4.9
51 B	101.6	10.9	9.7	91.5	.038	1405	6.2
52 B	101.6	11.5	9.7	82.5	.043	1193	4.7
53 B	101.6	11.0	9.7	91.5	.037	1392	6.4
54 B	101.6	11.3	9.7	91.5	.057	1193	4.0
55 T	101.6	10.8	9.6	60.0	.045	973	3.6
56 T	101.6	11.2	9.2	75.0	.051	1270	4.3
57 T	101.6	10.9	9.2	81.0	.041	1402	5.9
58 T	101.6	11.9	9.2				
59 B	101.6	10.6	9.2	57.0	.030	1025	5.9
60 B	101.6	10.7	9.0	61.5	.038	1118	5.3

Ave. S_T = 1182 ±161; Ave. S_B = 1217 ±138
Ave. E_T = 5.4 ±1.6; Ave. E_B = 5.2 ±1.0

Apr. 14, 1983 (-19°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
61 B	101.6	11.3	8.9	120.0	.064	1941	6.0
62 T	101.6	10.2	8.6	91.5	.043	1838	8.4
63 B	101.6	11.4	8.3	121.5	.066	2303	7.3
64 T	101.6	11.0	8.5	88.5	.054	1662	6.2
65 B	101.6	11.0	8.5	108.0	.064	2071	6.4
66 T	101.6	11.0	8.5	85.5	.050	1609	6.5
67 B	101.6	11.2	8.5	114.0	.066	1937	6.5

Ave. S_T = 1703 ±120; Ave. S_B = 2063 ±172
Ave. E_T = 7.0 ±1.4; Ave. E_B = 6.6 ±0.5
+Off center break exceeding 10 cm.

SHEET 3 (Cont.)
(UNSEEDED)

Apr. 14, 1983 (-19°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
68	T	107.5	10.6	12.8	22.7	.095	826	5.4
69	T	108.0	11.4	12.7	27.4	.189	947	5.8
70	T	107.5	10.7	12.1	19.9	.125	733	4.1
71	T	107.0	10.8	10.9	19.9	.122	973	5.6
72	T	107.0	11.5	11.0	23.6	.138	1069	5.4
73	T	107.0	10.9	11.0	19.9	.106	983	6.2
74	T	104.5	11.2	11.2	8.0	.042	351 #	5.4
75	B	99.0	10.5	11.5	24.6	.157	1030	3.7
76	B	107.5	11.5	11.5	29.8	.134	1238	6.2
77	B	107.5	11.5	11.6	24.1	.073	1002	9.0
78	B	BROKEN						
79	B	107.5	11.7	11.8	27.9	.131	1092	5.4
80	B	107.0	11.4	12.0	26.0	.115	996	5.5
81	B	100.5	10.6	11.8	30.7	.134	1293	5.2
82	B	106.5	11.5	11.8	37.8	.173	1479	5.5
83	B	106.0	10.8	11.5	29.8	.173	1306	4.9
84	B	106.5	11.1	11.4	24.1	.118	1047	5.9

Ave. S_T = 922 ±121; Ave. S_B = 1164 ±171
Ave. E_T = 5.4 ±0.7; Ave. E_B = 5.7 ±1.4

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
68	T	101.6	10.9	12.0	97.5		862	
69	T	101.6	10.4	12.0	108.8		1143	
70	T	101.6	10.4	11.6	105.0		1146	
71	T	101.6	11.0	11.6	96.0		1014	
72	T	101.6	11.3	11.4	127.5		1230	
73	T	101.6	11.5	11.5	116.2		959 +	
74	T	101.6	10.9	11.8	101.3		1040	
75	B	91.4	11.4	12.0	153.8		1305	
76	B	101.6	11.4	12.0	150.0		1423	
77	B	RECORDER MALFUNCTION						
78	B	81.3	11.4	12.0	198.8		1484	
79	B	101.6	11.1	12.0	120.0		1174	
80	B	101.6	11.0	12.0	161.2		1386	
81	B	101.6	11.6	12.0	161.2		1500	
82	B	101.6	11.0	11.8	131.3		1225	
83	B	101.6	11.1	11.6	150.0		1561	
84	T	101.6	11.0	11.4	108.8		1018	

Ave. S_T = 1064 ±120; Ave. S_B = 1382 ±137

Apr. 15, 1983 (-5°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
85	B	101.6	11.0	12.3	232.5		1527 +	
86	B	101.6	11.1	12.3	172.5		1394	

Ave. S_B = 1394

+Off center break exceeding 10cm
#Crack in ice sheet

SHEET 4
(SEEDED)

Apr. 25, 1983 (-1°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1	T	87.0	14.3	8.3	14.7	.092	761	5.0
2	T	86.5	13.5	8.8	14.6	.114	708	3.5
3	T	87.0	14.0	8.4	14.6	.108	754	4.2
4	B	88.0	13.1	8.4	12.9	.105	720	4.2
5	B	86.5	14.0	8.4	15.7	.114	808	4.2
6	B	84.0	13.9	8.5	18.2	.140	893	3.5
7	T	87.0	8.7	8.6	9.5	.092	752	4.8
8	T	87.0	8.8	8.8	11.7	.121	880	4.2
9	T	86.8	8.4	8.9	8.7	.086	667	4.4
10	B	86.5	8.7	8.9	10.4	.105	768	4.1
11	B	87.0	8.8	8.9	10.6	.105	777	4.2
12	B	87.0	8.8	8.8	10.2	.098	766	4.5

Ave. S_B = 754 ± 72; Ave. S_B = 789 ± 88
Ave. E_T = 4.4 ±0.5; Ave. E_B = 4.1 ±0.3

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1	T	81.3	13.9	8.4	112.5		1424	
2	T	81.3	13.7	8.5	115.5		1447	
3	T	81.3	13.9	8.5	129.0		1587	
4	B	81.3	13.4	8.8	87.0		1052	
5	B	81.3	13.9	8.8	90.0		1050	
6	B	81.3	13.8	9.0	97.5		1092	
7	T	81.3	9.3	9.0	104.9		1674	
8	T	BROKEN						
9	T	81.3	8.4	9.2	87.0		1511	
10	B	BROKEN						
11	B	81.3	8.2	9.3	57.0		879	
12	B	81.3	8.9	9.4	60.0		959	

Ave. S_T = 1529 ±103; Ave. S_B = 1006 ± 86

Apr. 27, 1983 (-1°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
13	B	91.4	10.4	7.9	62.2		1345	
14	B	91.4	9.8	7.7	54.8		1239	
15	B	91.4	9.9	8.0	61.7		1380	
16	T	91.4	10.2	8.2	104.5		2005	
17	T	91.4	10.0	8.2	104.5		2157	
18	B	91.4	9.7	8.2	62.2		1308	
19	T	91.4	10.2	8.3	107.0		2114	
20	B	91.4	10.0	8.5	26.4		1257	
21	B	91.4	9.8	8.7	59.7		1076	

Ave. S_T = 2092 ±78; Ave. S_B = 1268 ±108

SHEET 4 (Cont.)
(SEEDED)

Apr. 28, 1983 (-5°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
22	B	91.4	9.5	7.6	74.7	.065	1903	5.1
23	B	91.4	9.7	7.4	71.7	.043	1848	8.0
24	B	91.4	10.3	7.2	76.7	.057	2014	6.5
25	T	91.4	9.9	7.2	79.7	.062	1958	6.5
26	T	91.4	10.0	7.7	119.5	.090	2780	5.4
27	T	91.4	9.7	7.8	99.5	.078	2066	5.2
28	T	91.4	9.8	7.9	97.1	.075	2043	5.0
29	T	91.4	10.3	8.0	131.9	.076	2759	6.1

Ave. S_T = 2321 ±411; Ave. S_B = 1922 ± 85
Ave. E_T = 5.6 ±0.6; Ave. E_B = 6.5 ±1.5

Apr. 28, 1983 (-10°C) Cantilever

30	T	107.0	9.8	10.3	11.3	.089	687	5.7
31	T	107.5	10.0	10.2	12.3	.092	758	6.1
32	T	107.5	9.9	10.2	13.2	.130	816	4.7
33	B	107.5	10.5	10.0	13.7	.133	825	4.8
34	B	108.0	9.6	9.8	11.3	.121	793	5.1
35	B	106.5	10.4	9.8	12.3		771	
36	B	77.0	9.9	10.1	19.4	.080	869	4.3
37	B	77.5	9.8	10.1	16.1	.070	733	4.2
38	B	76.5	10.0	10.0	19.9	.086	897	4.1
39	T	77.0	9.9	10.0	16.5	.089	768	3.4
40	T	76.5	9.8	9.7	16.5	.089	811	3.6
41	T	76.5	10.0	9.7	15.6	.080	753	3.8

Ave. S_T = 766 ± 47; Ave. S_B = 815 ± 61
Ave. E_T = 4.5 ±1.1; Ave. E_B = 4.5 ±0.4

Apr. 29, 1983 (-10°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
42	T	RECORDER MALFUNCTION						
43	T	101.6	10.1	10.5	194.1	.067	2670	6.4
44	T	101.6	10.1	10.4	201.6	.102	2824	4.5
45	B	101.6	10.1	10.3	124.5	.056	1802	5.2
46	B	101.6	9.9	10.4	124.5	.087	1700	3.3
47	B	101.6	9.9	10.3	117.0	.061	1740	4.6
48	B	71.1	10.1	10.0	184.2	.065	1591	2.5
49	B	71.1	9.5	10.2	166.8	.064	1722	2.3
50	B	71.1	10.5	10.2	151.8		1486	
51	T	71.1	10.0	9.7	219.0	.073	2493	2.9
52	T	71.1	10.0	9.7	224.0	.027	2488	8.0
53	T	71.1	9.9	9.7	234.0	.025	2563	9.0

Ave. S_T = 2608 ±142; Ave. S_B = 1674 ±115
Ave. E_T = 6.2 ±2.5; Ave. E_B = 3.6 ±1.3

Apr. 29, 1983 (-10°C) Cantilever

54	T	110.0	12.0	9.0	11.8	.165	786	4.3
55	T	111.0	12.0	9.0	13.4	.178	902	4.6
56	T	110.0	11.5	9.0	13.6	.188	945	4.5
57	T	110.5	12.0	9.0	14.8	.197	986	4.6
58	B	109.5	11.5	9.0	9.8	.137	680	4.4
59	B	110.0	11.0	9.2	11.3	.156	788	4.4
60	B	109.5	11.5	9.1	11.3	.143	767	4.7
61		BROKEN						

Ave. S_T = 905 ± 86; Ave. S_B = 745 ± 57
Ave. E_T = 4.5 ±0.1; Ave. E_B = 4.5 ±0.2

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
54		BROKEN						
55	T	101.6	12.0	9.6	94.6	.045	1136	5.2
56	T	101.6	11.8	9.5	99.6	.056	1470	4.5
57	T	101.6	11.6	9.5	89.6	.045	1351	5.2
58	B	101.6	11.0	9.5	62.2	.030	963	5.6
59	B	101.6	12.3	9.5	72.2	.057	953	3.2
60	B	101.6	11.4	9.5	67.6	.038	1056	4.7
61		BROKEN						

Ave. S_T = 1319 ±169; Ave. S_B = 991 ± 57
Ave. E_T = 5.0 ±0.4; Ave. E_B = 4.5 ±1.2

SHEET 4 (Cont.)
(SEEDED)

May 2, 1983 (-19°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
62	T	110.5	11.2	9.4	15.1		993	
63	T	110.0	11.4	9.6	16.7	.171	1025	5.1
64	T	109.5	11.7	9.6	15.1	.156	903	4.8
65	T	109.5	11.8	9.8	16.6	.178	941	4.3
66	T	109.5	11.7	9.8	15.1	.133	866	5.3
67	B	111.0	11.6	9.9	13.6	.130	783	5.0
68	B	109.5	11.3	10.3	18.7	.184	1005	4.2
69	B	110.5	11.5	10.0	16.6	.159	936	4.8
70	B	110.0	10.7	10.0	12.8	.116	773	5.4

Ave. S_T = 946 ± 65; Ave. S_B = 874 ±115
Ave. E_T = 4.9 ±0.4; Ave. E_B = 4.9 ±0.5

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
62	T	101.6	11.2	9.8	141.9	.057	2126	6.1
63		RECORDER MALFUNCTION						
64		"	"					
65		"	"					
66		"	"					
67		"	"					
68	B	101.6	11.4	10.4	69.7	.022	912	6.2
69	B	101.6	11.5	10.4	93.6	.040	1191	4.6
70	T	101.6	11.2	10.5	114.5	.059	1449	3.9

Ave. S_T = 1788 ±479; Ave. S_B = 1052 ±197
Ave. E_T = 5.0 ±1.6; Ave. E_B = 5.4 ±1.1

May 2, 1983 (-19°C) Isothermal Simple Support

71	T	101.6	11.1	8.9	124.4	.057	2192	7.2
72	T	101.6	11.7	9.0	171.7	.075	2783	6.9
73	T	101.6	11.3	9.0	166.8	.071	2799	7.3
74	B	101.6	11.5	8.7	97.1	.038	1663	8.7
75	B	101.6	11.5	8.7	97.1	.048	1745	6.9
76	B	101.6	11.5	8.5	89.6		1692	
77	B	101.6	11.3	8.9	82.1	.033	1465	8.0
78	T	101.6	11.3	8.7	139.4	.041	2514	11.8

Ave. S_T = 2572 ±285; Ave. S_B = 1641 ±122
Ave. E_T = 8.3 ±2.3; Ave. E_B = 7.9 ±0.9

May 3, 1983 (-5°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
79	T	101.6	11.8	10.6	219.0	.097	2168	4.1
80	T	101.6	11.9	10.6	199.2	.070	2247	5.2
81	T	101.6	11.4	10.6	176.7	.054	2127	6.2
82	T	101.6	11.5	10.6	194.1	.056	2199	6.6
83	B	101.6	11.2	10.8	104.5	.027	1129	7.2
84	B	101.6	12.0	10.9	107.0		1185	
85	B	101.6	11.2	10.8	129.4	.033	1544	7.1
86	B	101.6	11.1	10.9	92.1	.030	1107	5.5
87	T	101.6	10.7	11.1	199.1	.051	2330	6.9

Ave. S_T = 2214 ± 78; Ave. S_B = 1241 ±204
Ave. E_T = 5.8 ±1.1; Ave. E_B = 6.6 ±1.0

May 4, 1983 (-1°C) Isothermal Simple Support

88	T	101.6	11.3	9.2	104.5	.049	1709	6.2
89	T	101.6	11.0	9.3	159.3	.064	2576	7.3
90	T	101.6	11.0	9.6	161.8	.078	2456	5.5
91		BROKEN						
92	B	101.6	10.5	10.0	77.2	.021	1167	9.2
93	B	101.6	10.9	10.0	87.1	.030	1269	5.7
94	B	101.6	10.5	10.0	82.1	.032	1108	6.3
95	T	101.6	10.8	11.0	117.5	.027	1406	7.8
96	B	101.6	10.1	11.1	102.1		1070	
97	T	101.6	10.4	10.6	134.4	.032	1749	8.7

Ave. S_T = 1979 ±509; Ave. S_B = 1154 ± 87
Ave. E_T = 7.1 ±1.3; Ave. E_B = 7.1 ±1.9

SHEET 5
(UNSEEDED)

May 18, 1983 (-1°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1 T	107.5	11.1	9.0	12.8	.121	897	6.4
2 T	108.5	11.0	9.2	15.9	.152	1088	6.1
3 T	107.5	11.2	9.2	12.1	.111	807	6.1
4 T	108.0	10.6	9.2	13.6		964	
5 T	106.5	11.8	9.2	18.7	.219	1174	4.4
6 T	108.0	10.8	9.2	15.1	.143	1051	6.2
7 B	107.0	11.0	9.2	14.4	.133	971	6.1
8 B	95.0	11.5	9.6	14.8		777	
9 B	104.0	11.0	9.4	20.6	.181	1297	5.5
10 B	103.5	10.5	9.6	18.5	.162	1165	5.3

Ave. S_T = 997 ±134; Ave. S_B = 1053 ±227
Ave. E_T = 5.8 ±0.8; Ave. E_B = 5.6 ±0.4

Parallel Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1 T	101.6	10.6	9.2	87.1	.033	1526	8.1
2 T	101.6	11.0	9.1	102.1	.054	1618	5.9
3 T	101.6	11.2	9.3	92.1	.032	1572	8.3
4 T	101.6	11.0	9.5	89.6	.032	1395	7.7
5 T	101.6	11.0	9.3	104.5	.035	1717	8.7
6	BROKEN						
7 B	101.6	11.4	10.0	112.0	.041	1204 +	
8 B	91.4	11.0	10.1	104.5		1321	
9 B	101.6	11.0	10.1	99.5	.076	1394	3.0
10 B	101.6	11.0	10.1	97.1		1361	

Ave. S_T = 1566 ±119; Ave. S_B = 1359 ±37
Ave. E_T = 7.7 ±1.1; Ave. E_B = 3.0

May 18, 1983 (-1°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
11 B	101.6	9.9	8.5	107.0	.070	2317	6.4
12 B	101.6	10.6	8.5	104.5		2121	
13 B	101.6	10.5	8.7	107.0	.062	1676 +	
14 T	101.6	10.3	8.5	89.6	.040	1509 +	
15 T	101.6	10.3	8.6	84.7	.035	1691	9.5
16 T	101.6	10.5	8.8	64.7	.030	1314	7.7
17 T	101.6	10.2	8.8	72.2	.038	1444	7.0
18 B	101.6	10.4	8.7	115.5	.059	2272	7.4
19 T	101.6	10.1	8.8	79.7	.035	1466	9.7

Ave. S_T = 1479 ±157; Ave. S_B = 2237 ±103
Ave. E_T = 8.5 ±1.3; Ave. E_B = 6.9 ±0.7

+Off center break exceeding 10cm.

May 19, 1983 (-5°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
21 T	BROKEN						
22 T	107.0	11.1	8.1	9.0	.118	776	6.2
23 T	107.5	11.0	8.0	10.2	.137	917	6.5
24 T	106.5	11.4	8.1	12.7	.175	1061	5.7
25 T	108.0	11.4	8.0	10.0	.140	872	6.1
26 B	107.5	11.1	7.8	12.3	.191	1150	6.0
27 B	107.5	10.9	7.7	13.6		1332	
28 B	102.5	11.5	7.9	18.1	.222	1524	6.1
29 B	105.0	11.6	7.8	14.7	.261	1282	4.8
30 B	107.5	11.1	7.8	12.8	.194	1195	6.1

Ave. S_T = 907 ±119; Ave. S_B = 1297 ±146
Ave. E_T = 6.1 ±0.3; Ave. E_B = 5.8 ±0.6

Parallel Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
21 T	101.6	10.7	8.6	58.7	.029	1190	7.7
22 T	101.6	11.4	8.4	73.7	.037	1289	7.7
23 T	101.6	11.5	8.5	80.6	.035	1532	8.4
24 T	101.6	11.3	8.4	69.7	.029	1388	9.3
25 T	101.6	11.3	8.4	77.2	.033	1529	8.9
26 B	101.6	10.9	8.4	51.8	.022	1089	9.2
27 B	101.6	11.0	8.0	56.7	.029	1090	9.0
28	BROKEN						
29 B	101.6	11.3	7.9	83.6	.052	1528	7.4
30 B	101.6	11.7	7.8	84.6	.051	1865	7.7

Ave. S_T = 1386 ±150; Ave. S_B = 1393 ±376
Ave. E_T = 8.4 ±0.7; Ave. E_B = 8.3 ±0.9

May 19, 1983 (-5°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
31 T	101.6	11.5	10.3	161.3	.041	1974	8.0
32 T	101.6	12.0	10.3	151.8	.038	1551	7.8
33 T	101.6	11.3	10.1	131.9	.037	1778	8.0
34 T	101.6	10.9	10.3	156.8	.045	2093	7.6
35 B	101.6	11.5	10.0	134.4	.049	1816	6.1
36 B	101.6	11.3	10.0	119.5	.033	1478	8.2
37 B	101.6	11.4	10.0	149.5	.037	1930	9.2
38 B	101.6	11.9	10.0	161.8	.040	2060	8.8
39 B	101.6	11.1	10.1	156.8	.041	1974	8.5

Ave. S_T = 1849 ±237; Ave. S_B = 1852 ±227
Ave. E_T = 7.9 ±0.2; Ave. E_B = 8.2 ±1.2

SHEET 5 (Cont.)
(UNSEEDED)

May 20, 1983 (-19°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
41	T	108.0	11.6	10.8	18.5	.099	865	6.3
42	T	106.5	11.7	10.5	18.5	.083	895	7.8
43	T	105.0	11.7	10.4	27.4	.181	1338	5.2
44	T	106.0	11.5	10.4	8.5	.064	426	4.8
45	T	106.5	11.2	10.5	23.2	.149	1175	5.7
46	T	BROKEN						
47	B	107.0	11.0	10.3	18.9	.135	1019	5.6
48	B	98.0	10.6	10.3	22.2	.159	1138	4.5
49	B	96.5	11.0	10.0	17.0	.124	877	4.4
50	B	99.0	10.6	10.0	18.9	.146	1038	4.6
51	B	106.5	11.8	9.9	18.0	.137	973	5.5
52	B	105.0	11.0	10.0	23.6	.188	1327	5.2

Ave. S_T = 940 ±348; Ave. S_B = 1062 ±155
Ave. E_T = 6.0 ±1.2; Ave. E_B = 5.0 ±0.5

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
41	T	101.6	11.8	10.9	144.4		1602	
42		BROKEN						
43	T	101.6	10.7	10.4	122.0		1641	
44	T	101.6	11.6	10.9	114.5		1305	
45	T	101.6	10.9	10.8	109.5		1326	
46	T	101.6	11.4	10.8	122.0		1435	
47		BROKEN						
48	B	101.6	11.0	10.7	104.5		1159	
49		BROKEN						
50	B	91.4	10.7	10.1	112.0		1381	
51	B	101.6	11.8	10.5	114.5		1381	
52	B	101.6	10.6	10.4	102.1		1396	

Ave. S_T = 1462 ±155; Ave. S_B = 1329 ±114

May 20, 1883 (-19°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	S(kPa)
53	T	101.6	10.8	10.2	134.4	1855
54	T	101.6	11.1	9.8	151.8	2010
55	T	101.6	11.4	9.7	151.8	2186
56	T	101.6	10.9	10.2	149.4	2035
57	B	101.6	11.2	9.6	161.8	2252
58	B	101.6	11.6	10.1	161.8	2111
59	B	91.4	11.4	10.3	196.6	2153
60	B	101.6	10.9	10.0	149.4	1874
61	B	101.6	11.5	10.5	154.3	1741
62	B	101.6	10.9	10.4	166.8	1743

Ave. S_T = 2022 ±136; Ave. S_B = 2026 ±211

May 23, 1983 (-10°C) Cantilever

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
63	T	107.5	11.5	11.0	21.3	.124	966	5.5
64	T	107.0	11.0	10.8	25.1	.143	1228	6.1
65	T	107.0	11.5	10.8	24.1	.137	1131	5.8
66	T	107.0	11.3	10.2	20.8	.124	1113	6.7
67	T	107.5	11.5	10.8	26.0	.152	1225	5.7
68	B	106.0	11.8	10.5	24.6	.149	1178	5.6
69	B	106.0	11.8	10.5	29.3	.172	1404	5.8
70	B	102.5	11.4	10.4	23.6	.149	1155	5.2
71	B	101.5	11.5	11.4	27.0	.178	1293	3.7
72	B	103.5	11.0	10.2	19.4	.133	1031	5.4

Ave. S_T = 1133 ±107; Ave. S_B = 1212 ±142
Ave. E_T = 6.0 ±0.5; Ave. E_B = 5.1 ±0.8

Parallel Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
63	T	101.6	10.6	10.9	122.0	.037	1511	6.2
64	T	101.6	11.4	10.5	82.1		1043 +	
65	T	101.6	11.7	10.9	126.9	.025	1428	8.5
66	T	101.6	11.4	11.1	122.0	.032	1359	6.3
67	T	101.6	11.5	10.9	144.4	.043	1642	5.8
68	B	101.6	11.5	10.9	149.4	.035	1616	7.4
69	B	101.6	11.8	10.7	174.2	.038	1991	8.1
70	B	101.6	11.6	10.7	151.8	.03[illegible]	1772	8.2
71	B	101.6	11.4	10.5	112.0	.027	1214	8.1
72	B	101.6	11.3	10.4	144.4	.035	1831	8.3

Ave. S_T = 1485 ±122; Ave. S_B = 1685 ±295
Ave. E_T = 6.7 ±1.2; Ave. E_B = 8.0 ±0.4

May 23, 1983 (-10°C) Isothermal Simple Support

BEAM		L(cm)	w(cm)	h(cm)	P(kg)	S(kPa)
73	T	101.6	11.4	12.9	256.4	2072
74	T	101.6	11.3	12.8	208.4	1735
75	T	101.6	11.8	12.9	255.7	1769
76	B	101.6	10.9	13.1	309.4	2523
77	B	101.6	11.4	12.5	246.2	1998
78	B	101.6	11.6	12.3	252.5	2206
79	T	101.6	11.9	12.1	230.4	2058
80	T	101.6	11.3	13.9	284.1	2007
81	B	101.6	11.5	14.0	249.4	1682
82	T	101.6	11.2	13.6	255.7	1909

Ave. S_T = 1925 ±146; Ave. S_B = 2102 ±354

+Off center break exceeding 10cm.

SHEET 6
(SEEDED-UNSEEDED)

June 7, 1983 (-5°C) Cantilever - Seeded
Test #1 - 10: No results because of defective load cell

June 7, 1983 (-5°C) Isothermal Simple Support - Seeded

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
11 T	91.4	11.1	8.2	112.0	.065	1783	5.3
12 T	91.4	9.6	8.1	97.1	.056	2130	6.4
13 T	91.4	11.7	8.2	146.9	.076	2577	5.6
14 T	91.4	9.3	8.0	99.6	.062	2306	6.3
15 T	91.4	9.7	8.2	99.6	.059	2131	5.9
16 T	91.4	10.2	7.7	94.6	.059	2174	6.5
17 T	91.4	10.3	7.4	89.6	.059	1885	6.8
18 B	91.4	10.7	8.0	62.2	.035	1298	6.1
19 B	91.4	9.5	8.2	63.7	.030	1393	7.5
20 B	91.4	10.[illegible]	8.2	75.7	.041	1551	6.1
21 B	91.4	11.3	8.2	85.6	.037	1564	7.0
22 B	91.4	11.2	8.0	77.2	.043	1492	5.9
23	BROKEN						
24 B	91.4	10.2	7.8	89.6	.041	1339	8.5
25 T	91.4	11.7	8.0	123.5	.070	2285	5.5
26	RECORDER MALFUNCTION						
27 T	91.4	10.6	7.8	124.5	.076	2687	6.1
28 B	91.4	12.0	8.1	59.7	.025	1076	6.9
29 T	81.3	11.0	7.9	117.0		2120	

Ave. S_T = 2208 ±276; Ave. S_B = 1388 ±171
Ave. E_T = 6.0 ±0.5; Ave. E_B = 6.9 ±0.9

June 8, 1983 (-5°C) Isothermal Simple Support - Unseeded

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
44 B							
45 B	91.4	12.1	11.1	171.7	.027	1598	7.2
46 B	91.4	12.5	10.7	248.9	.038	2367	8.0
47 B	91.4	8.7	10.7	126.9	.028	1749	9.7
48 B	91.4	12.0	10.8	201.6	.035	1734	7.1
49 B	91.4	9.4	10.6	141.9	.032	1841	7.5
50 T	91.4	12.0	10.8	186.7	.032	1855	6.7
51 T	91.4	9.3	10.7	144.4	.033	1601	7.1
52 T	91.4	12.7	10.7	149.3	.025	1433	7.1
53 T	91.4	9.3	10.5	164.3	.040	2135	7.2
54 T	91.4	10.5	10.5	161.8	.033	1933	7.5
55 T	91.4	9.4	11.2	149.3		1752	
56 B	91.4	10.9	11.1	206.6	.048	2099	5.4
57 B	91.4	9.2	11.0	179.2	.035	2214	7.8
58 B	91.4	12.7	11.1	241.4	.041	2019	6.3
59 B	91.4	9.0	11.0	164.3	.037	2078	7.0
60 T	91.4	11.5	10.8	206.6		2124	
61 T	91.4	9.3	10.7	161.8	.049	2032	5.4
62	BROKEN						
63 T	91.4	11.3	10.5	159.3	.030	1772	7.5
64 T	91.4	11.7	10.5	159.3	.035	1713	6.3
65 T	101.6	11.4	10.1	122.0	.033	1620	8.0
66 B	101.6	11.4	10.0	191.7	.061	2581	7.1

Ave. S_T = 1815 ±225; Ave. S_B = 2028 ±307
Ave. E_T = 7.0 ±0.8; Ave. E_B = 7.3 ±1.1

June 7, 1983 (-5°C) Cantilever - Unseeded

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
30 T	DEFECTIVE LOAD CELL						
31 T	" " "						
32 B	105.5	11.2	9.9	15.5	.118	877	5.6
33 B	106.0	11.6	10.0	19.4	.143	1031	5.6
34 Bm	99.5	13.1	10.2	30.3	.178	1299	4.7
35 Bm	91.0	11.7	10.4	27.4	.178	1159	3.5
36 Bm	101.5	14.0	10.4	34.5	.207	1360	4.4
37 Bm	99.0	10.9	10.5	22.7	.149	1099	4.6
38 T	96.5	12.6	10.5	DEFECTIVE LOAD CELL			
39 T	100.5	12.2	10.3	" " "			
40 T	103.0	12.7	10.2	" " "			
41 T	99.0	13.0	10.4	" " "			
42 T	102.0	13.4	10.2	" " "			
43 T	94.0	10.0	10.3	" " "			

Modified - Ave. S_B = 1229 ±121
Ave. E_B = 4.3 ±0.5

Unmodified - Ave. S_B = 954 ±109
Ave. E_B = 5.6

m = modified, 5 cm. radii at butt end

June 9, 1983 (-5°C) Cantilever - Unseeded

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
67 Tm	105.0	13.5	11.8	33.4	.143	1096	4.8
68 Tm	98.5	13.6	11.5	39.1	.159	1258	4.6
69 Bm	99.0	13.8	11.4	35.8		1165	
70 Bm	91.0	11.7	11.0	34.3	.197	1309	3.3
71 Bm	107.0	13.0	10.5	29.3	.184	1287	5.1
72 Bm	96.5	10.7	10.4	24.3	.172	1198	4.2
73 Tm	106.0	10.9	10.4	21.5	.152	1145	5.4
74 T	106.0	11.3	10.2	16.7	.127	885	5.1
75 T	98.0	12.1	9.9	17.6	.140	857	4.0

Modified - Ave. S_T = 1166 ± 83; Ave. S_B = 1240 ± 69
Ave. E_T = 4.9 ±0.4; Ave. E_B = 4.2 ±0.9

Unmodified - Ave. S_T = 871 ± 20
Ave. E_T = 4.6 ±0.8

m = modified, 10 cm. radii at butt end

SHEET 7
(SEEDED)

July 11, 1983 (-5°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1 Tm	113.0	13.3	9.6	27.2		1475	
2 Tm	120.0	15.0	9.6	18.0		919	
3 Tm	118.5	15.0	9.6	22.7		1144	
4 Tm	117.0	14.0	9.6	25.1		1338	
5 Tm	118.0	14.0	9.6	20.8		1119	
6 Tm	118.0	14.0	9.6	23.6		1269	
7 Tm	118.0	15.0	9.6	26.0		1311	
8 T	107.0	11.0	8.4	9.5		770	
9 T	107.5	11.4	8.4	9.0		707	
10 T	108.5	9.8	8.5	13.7		1234	
11 B	107.5	11.2	8.7	9.9		738	
12 B	108.0	10.7	8.8	10.9		835	
13 B	108.0	11.3	9.0	11.8		819	
14 B	108.0	11.0	9.1	12.3		857	
15 Bm	111.0	14.0	9.2	18.9		1041	
16 Bm	106.5	11.4	9.2	10.9		707	
17 Bm	108.5	12.7	9.2	16.5		979	
18 Bm	107.0	11.3	8.8	13.7		985	
19 Bm	112.0	14.3	8.3	10.4		695	
20 Bm	111.0	13.9	8.3	15.1		1029	
21 Bm	108.0	11.6	8.5	9.9		750	
22 Bm	101.0	11.5	8.7	11.3		771	
23 Bm	110.0	12.9	8.5	12.3		854	
24 Tm	109.5	11.7	8.5	16.1		1226	
25 Tm	105.5	13.0	8.2	14.2		1008	
26 Tm	103.0	12.0	8.0	17.0		1341	
27 Tm	107.0	12.4	8.0	14.2		1126	
28 Tm	109.5	12.2	7.8	12.8		1116	

Modified - Ave. S_T = 1199 ±157
Ave. S_B = 868 ±142

Unmodified - Ave. S_T = 904 ±288
Ave. S_B = 812 ± 52

m = modified, 10cm. radii at butt end

July 12, 1983 (-5°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
29 B	101.6	12.7	9.6	109.5	.042	1470	6.0
30 B	101.6	11.5	9.6	117.0	.045	1721	6.6
31 B	101.6	12.6	10.0	117.0	.045	1456	5.3
32 T	101.6	10.5	9.6	171.7		2298	
33 T	101.6	11.7	10.0	99.6	.035	1341	6.3
34 T	101.6	12.2	10.0	164.3	.056	2081	6.2
35 T	101.6	[illegible]	9.9	243.9	.080	2725	5.8
36 T	91.4	[illegible]	9.3	176.7	.053	2367	6.5
37 T	81.3	11.5	9.6	221.5		2571	
38 T	81.3	11.9	10.1	189.2		1906	

Ave. S_T = 2184 ±463; Ave. S_B = 1549 ±149
Ave. E_T = 6.2 ±0.3; Ave. E_B = 6.0 ±0.7

July 14, 1983 (-10°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
39 T	101.6	12.4	8.5	179.2	.090	3069	6.7
40 T	101.6	12.5	8.5	119.5	.058	2058	6.9
41 T	101.6	11.3	9.0	139.4	.067	2352	6.5
42 T	101.6	11.2	9.2	149.3	.070	2428	6.6
43 B	101.6	10.7	8.3	74.7		1597	
44 T	101.6	11.0	8.5	119.5	.080	2328	5.7
45 B	101.6	11.5	8.6	72.2	.045	1349	5.6
46 B	101.6	11.2	8.8	94.6	.048	1708	6.6
47 B	101.6	11.1	8.9	82.1	.038	1473	7.1
48 B	101.6	10.8	8.6	84.6	.048	1663	6.6
49 B	101.6	10.5	8.6	89.6	.048	1804	7.2
50 T	101.6	11.5	8.4	124.4		2373	
51 T	101.6	11.4	8.2	122.0	.074	2462	6.7
52 T	101.6	10.6	8.0	92.1	.048	2115	9.1
53 T	101.6	12.9	9.2	159.3	.064	2255	6.4
54 T	101.6	10.7	9.3	107.1	.051	1803	6.3
55 B	101.6	12.2	9.2	94.6	.038	1444	6.7
56 B	101.6	10.4	9.0	84.6	.038	1577	7.6
57 B	101.6	11.8	8.5	87.1	.038	1608	8.1
58 B	101.6	11.1	8.9	94.6	.042	1685	7.4
59 T	91.4	11.5	8.8	149.3	.046	2316	7.8
60 B	71.1	11.7	8.4	129.4	.027	1679	6.1
61 T	71.1	11.5	8.7	196.6	.038	2400	6.0

Ave. S_T = 2329 ±299; Ave. S_B = 1598 ±132
Ave. E_T = 6.8 ±0.9; Ave. E_B = 6.9 ±0.7

SHEET 8
(SEEDED)

November 7, 1983 (-1°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1 T	105.0	9.4	10.4	10.1	.092	612	4.7
2 T	105.5	10.1	10.4	12.7	.102	723	5.0
3 T	106.0	10.6	10.5	13.5	.102	720	5.0
4 T	106.0	10.1	10.5	12.3	.102	690	4.8
5 B	105.5	10.6	10.5	12.6	.099	666	5.0
6 B	104.5	10.1	10.6	14.4	.122	782	4.4
7 B	104.5	10.1	10.8	16.3	.137	852	4.2
8 B	104.5	10.2	10.6	12.4	.099	663	4.6
9 B	105.0	10.2	10.8	15.6	.127	810	4.3
10	BROKEN						
11 Bm	105.0	10.5	10.5	18.1	.111	965	3.5
12 Bm	111.0	14.0	10.5	12.4		523	
13 Bm	100.0	11.5	10.4	20.4	.218	966	2.8
14 Bm	110.0	13.3	10.1	19.1	.226	905	3.2
15 Bm	110.0	12.6	10.0	13.3		684	
16 Tm	105.0	11.5	9.9	18.1	.229	988	3.2
17 Tm	105.0	10.1	9.6	13.8	.213	913	3.3
18 Tm	105.0	12.5	9.3	13.5	.203	771	3.0
19 Tm	105.0	11.7	9.2	13.7	.196	853	3.5
20 Bm	106.0	10.8	9.0	13.8	.232	981	3.5
21 Bm	113.0	15.5	8.4	13.7	.248	833	3.4

Unmodified - Ave. S_T = 686 ± 52; Ave. S_B = 754 ± 86
Ave. E_T = 4.9 ±0.2; Ave. E_B = 4.5 ±0.3

Modified - Ave. S_T = 881 ± 92; Ave. S_B = 836 ±173
Ave. E_T = 3.2 ±0.3; Ave. E_B = 3.3 ±0.2

m = modified, 10cm. radii at butt end

November 8, 1983 (-1°C) Cantilever

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
A T	104.5	10.4	8.0	7.3	.102	669	6.0
B T	104.0	10.2	8.0	7.3	.114	641	5.4
C T	104.0	9.4	8.0	7.3	.117	736	5.7
D T	103.5	10.4	8.1	8.4	.127	747	5.2
E T	105.0	10.1	8.2	8.4	.127	761	5.4
F B	104.5	10.2	8.3	7.8	.120	685	5.0
G B	105.5	10.4	8.6	8.4	.124	675	4.7
H B	105.0	10.1	8.7	8.8	.124	709	4.8
I B	105.0	10.0	8.9	11.4	.152	891	4.8
J B	106.0	10.3	8.9	9.9	.143	756	4.5
K Bm	92.0	10.4	9.3	15.5	.225	931	2.5
L Bm	106.0	11.1	9.0	14.8	.210	1023	4.1
M Bm	109.5	13.5	8.9	12.9	.203	775	3.4
N Bm	104.5	12.6	8.5	13.4	.213	902	3.6
O Tm	106.0	11.2	8.4	13.9	.264	1097	3.7
P Tm	112.0	14.0	8.3	13.9	.251	950	3.8
Q Tm	110.0	11.8	8.3	11.1	.210	881	4.1
R Tm	109.5	12.8	8.1	11.1	.254	1038	3.3
S Tm	111.0	13.5	8.1	14.3	.279	1054	3.8
T Tm	111.0	13.6	8.0	13.7	.260	1029	4.1
U	BROKEN						
V	BROKEN						
W Tm	107.5	11.7	7.2	22.0		1032	

Unmodified - Ave. S_T = 711 ± 53; Ave. S_B = 743 ± 88
Ave. E_T = 5.5 ±0.3; Ave. E_B = 4.8 ±0.2

Modified - Ave. S_T = 1012 ± 72; Ave. S_B = 908 ±102
Ave. E_T = 3.8 ±0.3; Ave. E_B = 3.4 ±0.7

m = Modified, 10cm radii at butt end

November 8, 1983 (-1°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
1 T	101.6	10.5	10.5	126.9		1624	
2 T	101.6	10.0	10.7	124.5	.032	1305	8.2
3	BROKEN						
4 B	101.6	10.0	10.4	72.2	.024	1034	6.9
5 B	101.6	10.7	10.5	67.2		917	
6 B	101.6	9.9	10.5	66.7	.034	979	4.4
7	BROKEN						
8 T	101.6	9.8	10.7	149.4	.064	2053	5.0
9 T	101.6	10.2	10.7	156.8		1657	
10	BROKEN						
11	BROKEN						
12 B	101.6	10.6	10.5	82.1	.025	1075	6.9
13 B	91.4	11.4	10.2	77.2	.024	930	
14 T	101.6	11.2	10.1	109.5	.041	1501	6.0
15 T	101.6	11.5	10.0	102.1	.041	1395	5.6
16 B	101.6	11.6	9.9	74.7	.025	1055	6.8
17 B	101.6	10.5	9.7	59.7	.022	969	7.3
18 T	101.6	11.2	9.6	94.6	.024	1442	10.2
19 T	101.6	11.2	9.3	99.5	.051	1610	5.6
20 B	101.6	10.8	9.2	69.7	.027	1214	7.9
21 B	101.6	11.1	8.5	69.7	.035	1316	7.5

Ave. S_T = 1573 ±229; Ave. S_B = 1054 ±133
Ave. E_T = 6.8 ±2.0; Ave. E_B = 6.8 ±1.1

November 9, 1983 (-1°C) Isothermal Simple Support

BEAM	L(cm)	w(cm)	h(cm)	P(kg)	d(cm)	S(kPa)	E(GPa)
A T	101.6	9.8	8.1	52.3	.035	1253	7.4
B T	101.6	9.9	8.0	58.7	.038	1458	7.8
C T	101.6	9.8	8.0	61.7	.048	1557	6.6
D T	101.6	10.2	8.1	60.8	.044	1415	6.6
E T	101.6	10.2	8.3	66.7	.038	1501	7.7
F B	101.6	9.9	8.5	42.8	.030	976	6.0
G B	101.6	10.3	8.8	62.7	.032	1254	7.2
H B	101.6	9.7	9.0	51.8	.025	1061	7.5
I B	101.6	9.7	9.1	49.8	.025	1002	7.0
J B	101.6	10.3	9.1	52.8		1001	
K B	91.4	10.1	9.6	67.2		1029 +	
L	BROKEN						
M B	101.6	10.3	9.5	64.7	.027	1113	7.0
N B	101.6	11.9	8.8	66.7	.029	1160	7.3
O T	101.6	10.5	8.8	69.7	.029	1321	8.6
P T	101.6	10.2	8.9	69.7	.049	1367	5.1
Q T	101.6	10.4	8.9	74.7	.045	1432	5.8
R T	101.6	10.6	8.6	67.7	.048	1243	5.4
S	BROKEN						
T	BROKEN						
U B	101.6	11.3	8.4	51.8	.025	1053	8.3
V B	101.6	10.6	8.2	47.8	.029	1087	7.3
W B	101.6	9.6	7.5	39.8	.035	1195	7.2

Ave. S_T = 1394 ±108; Ave. S_B = 1090 ± 91
Ave. E_T = 6.8 ±1.2; Ave. E_B = 7.2 ±0.6

+ Off center break exceeding 10cm.

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Gow, Anthony J.

Temperature and structure dependence of the flexural strength and modulus of freshwater model ice / Anthony J. Gow, Herbert T. Ueda, John W. Govoni and John Kalafut. Hanover, N.H.: U.S. Army Cold Regions Research and Engineering Laboratory; Springfield, Va.: available from National Technical Information Service, 1988.

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